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# Investigating the Effects of Government Subsidies on Manufacturers and Remanufacturers in Green Economy- Based on Two-Period Game Theory

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**Abstract**
Under resource constraints and environmental degradation, remanufacturing industry has been increasingly promoted to fasten Green Economy and sustainable growth of the whole society, which require government subsidies. While the quantitative influence and the allocation of the subsidies are still uncertain. To help the government maximize the utility of these subsidies, it seems necessary to measure the outcomes in different cases, and the observation period should be expanded. Based on these and our prior research of their relationship, a two-period game model between manufacturers and remanufacturers is established, which shows their entrance orders. The three cases-a case without subsidies, a case with subsidies for the remanufacturer, and a case with subsidies for the consumer who selects the remanufactured product are compared based on this dynamic game model. The results indicate that government subsidies promote lower retail prices for both products; remanufactured products have greater price reductions when governments subsidize the remanufacturer, and brand-new products exhibit lower prices after consumer subsidies; vicious competition may occur if the government over-subsidizes consumers because brand-new product manufacturers will adopt a cut-throat strategy to compete with the lower-priced remanufactured products to survive the competition. Government subsidies cannot exceed the value of A/2 and recovery rate should be proper. By having clear view of these results, government subsidies will produce optimal economic outcomes for remanufacturing industry.
**Keywords:** game theory; government subsidies; remanufacturing; Green Economy; Sustainability

## 1. Introduction

With the increment of population, resource constraints have become serious obstacles for sustainable development of enterprises, relevant industries and the whole society. Some countries which rely heavily on natural resources have suffered considerable losses in recent age. This is more significant in China and other developing countries in rapid Industrial Age Qinghua, et al. (2014),. Since the 1980s, China has had an economic miracle of 7 per cent annual growth. People's increasing material demand and the development of supply capacity has not been a good match. After large-scale production encounters resource bottleneck, it appeals "promoting circular production of enterprises, circular combination of industries and constructing circular industrial system".

From environmental perspective, considerable enterprises and industries have to bear inescapable responsibility for this phenomenon. Traditionally, "Treatment after Pollution" was an important guideline in the balance game between economic development and environmental protection in China, as the enterprises adhere to the principle of "all is fish that comes to net". Many manufacturing enterprises and industries are in pursuit of maximize economic efficiency, which at the same time failed to protect the social benefits. For instance, manufacturing industries produce industrial wastes and poisonous gas annually

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as the results of meeting temporary market demands (Nordhaus, 1992; Garriga and Domènec, 2013).

As a result of resource shortages and environmental degradation, governments, businesses and the public are devoting increasing attention to resource sustainability. The purpose of this sustainability refers to the development situation that not only meets the needs of the present generation, but also do no harm to the production and life of future generations. An effective method of positively impacting resource sustainability is through remanufacturing, a process that saves energy and reduces emissions (Hatcher, Ijomah and Windmill, 2011).

Remanufacturing is the key factor of sustainable development and Green Economy. The conference on the Human Environment held in 1972 specially adopted the Declaration on the Human Environment, which has become a solemn declaration of human environmental protection for the whole world and has played an important role in guiding all mankind to effectively protect the ecological environment. In fact, the Green Economy is not only the solutions on environmental pollution, but also the consideration of restoration and creation of ecological resources (Garriga and Domènec, 2013). E.g., in 2015 the national financial subsidy policy for remanufactured products in China led by the National Development and Reform Commission has been fully launched after a three-month trial operation. Users who purchase remanufactured products of designated models can enjoy certain rate of state financial subsidies. The relevant policy should closely combine population, resources, environment and development, coordinate the relations between pollution and protection, production and consumption, economy and environment, development and consumption, and fundamentally solve the environmental and ecological problems (Abdulrahman, et al., 2014)

To attain these objectives, governments are developing several relevant policies and launching activities to promote the remanufacturing industry. For example, the State Council of China has declared that the central government should encourage the development of the remanufacturing industry with respect to a circular economy; thus, in 2009, the council launched related laws toward this end. Then the government has promoted Green Supply Chain and appealed the public to fill the relevant industries with Green Energy. Although the thirteen-five project (2016 to 2020) in China emphasizes the importance of sustainable development, which is regarded as the necessary path of upgradings, and provides several subsidies to promote the selection of remanufactured products, the sales of remanufactured products in the market have fallen short of expectations (Cui et al., 2017a; Cui et al., 2017b). Then Chinese government has focused its attention on developing the automobile remanufacturing industry and has thus offered substantial subsidies to encourage the selection of remanufactured components. However, there are debates about who the government should subsidize (Zhao and Chen, 2019). The government usually has two directions: remanufactures and consumers in the market. Thus, this study aims to develop a two-period game model based on government subsidies, and then determine the subsidy object by comparing the three cases (a case of the government not adopting a subsidy, a case of the government providing subsidies to remanufacturers and a case of the government providing subsidies to consumers).

Government subsidies are regarded as a fiscal policy tool (Schwartz and Clements, 2010). To understand how government subsidies affect the development of remanufacturers, two kinds of previous studies can be referenced. One investigates how government subsidies can effectively promote the development of a new industry. Martin et al. (2008) adopted a clean development mechanism tax/subsidies optimization model to identify the dynamics of launching a government tax/subsidies program. Based on this model, the economic and environmental benefits under a clean development mechanism are considered when searching for an optimal alternative to a tax/subsidies return. Toshimitsu (2016) used an environmentally differentiated duopoly model to analyze a consumer-based environmental subsidies policy, and the results indicate that a consumer-based environmental subsidies policy cannot reach social optimization if the marginal social value of environmental damage exceeds a certain value. In other words, the optimal alternative depends on the degree of the marginal social value. Lee and Cin (2010) conducted an empirical study, the results of which indicate that the research and development of subsidies enables an approach that involves sharing risks and motivates the research and development of investments by small- and medium-sized enterprises in developing countries. Dahl and Stokkereit (2019) studies a duopoly investment model with uncertainty, and in this model two enterprises get a monopoly benefit and information benefit of cost-reduction respectively in a certain period. Although evidence is not available to support the bundled effects of government subsidies, these results show that government subsidies may help to overcome the barriers to research and development; in addition, the government shares the risk in the development of small- and medium-sized enterprises by granting subsidies (Chau et al., 2005;Zhang et al., 2011; Change et al., 2013). Subsequently, Hong et al. (2012) assessed the government subsidies effect in a vehicle-to-grid system using a survey instrument, and the results indicate that if the Korean government provides annual subsidies amounting to one trillion Korean yuan, it can generate 1.94 trillion yuan for social welfare and grow the profits of the vehicle-to-grid service to 1.98 trillion yuan.

The other domain focuses on finding an optimal method of assisting the government in allocating subsidies to motivate the development of the remanufacturing industry. Kreps (1990) and Dockner et al. (2000) first introduced the concept of differential games, which are characterized by both multiperiod and strategic decision making, and this concept is used for economic modeling and analyzing management issues. Mitra et al. (2008) employed a two-period game model to develop three types of models. These models include giving subsidies to manufacturers, giving subsidies to remanufacturers and sharing the subsidies between manufacturers and remanufacturers. Tong, Zhizhen and Shou (2017) have compared the consequences of tax and subsidies. They have concluded that trade-in subsidy can encourage customers to replace their existing products with new and remanufactured products; remanufacture subsidy is beneficial to manufacturer and can further promote remanufacturing development. The results of their study indicate that subsidies enable the strengthening of remanufacturing activities and enhance the profits of remanufacturers. Furthermore, allocating all subsidies to remanufacturers would motivate manufacturers to acquire a share of subsidies so that manufactures would then produce products that are more suitable for remanufacturing. Wang et al. (2014) investigated the effect of subsidies policies on the Chinese recycling and remanufacturing industries using a dynamic system approach and introduced four types of subsidies policies: initial subsidies, recycling subsidies, research and development subsidies and production subsidies. Park (2015) analyzes the efficiency and productivity change within government subsidy recipients of a national technology innovation research and development (R&D) program.

These policies have different motivational objectives and features. For example, initial subsidies play an aggressive role in improving remanufacturing activities and can be launched in the initial stage of industry development. Liu et al. (2016) adopted a quality-based price competition model to evaluate the electrical waste and recycling of electronic equipment in a dual channel environment that includes formal and informal sectors, and the findings indicated that the marginal effect of government subsidies is not significant because of the higher quality level of waste. Once government subsidies were not substantial to balance the high quality of waste, the informal sector was able to generate a competitive advantage. Zhao et al. (2018) and Cui et al. (2017b) developed a decision-making model to consider customer preferences by investigating the relationship between remanufactured products and government subsidies, and their findings indicate that if remanufacturers share subsidies with customers, customers can be motivated to select remanufactured products and expand the market size by generating profits.

This study is an extension of the research on government subsidies for remanufacturing supply chains. The paper by Mitra et al. (2008) is related to ours but differs in that it covers government subsidies to customers and in its analysis process. Their study compares the three cases (giving subsidies to manufacturers, giving subsidies to remanufacturers and sharing subsidies between manufacturers and remanufacturers) through simulation analysis, but in our study, a general conclusion is obtained. Moreover, compared with Qinghua, et al. (2017), the recovery rates of the used product and the government subsidies strategies affecting the environment are also comparatively analyzed. We consider balancing the economic production with social strengths and promote Green Economy based on two-period model.

To achieve the research goals, the remainder of this paper is organized as follows. Section 2 introduces the model formulation. This part will show the model assumptions, notations, and modeling concepts. Three cases mentioned above will divide the functions and restraint conditions into three forms. Section 3 analyzes the models applied to the three cases. The discussion and conclusion are explored in Section 4.

## 2 Model

### 2.1 Model Assumptions

As shown in Figure 1, this study adopts a two-period game theory to analyze the impact of government subsidies on consumers, remanufactures and brand-new product manufacturers by comparing the models under different situations. In this paper, "two-period" represents the order of entry into the product market for different objectives. Remanufacturers are the latter one who make financial decisions and obtain resources based on the conditions of manufactures.

Our past paper hypotheses: at the same stage, the wholesale prices of new products per unit and remanufactured products per unit are determined by the original manufacturers and remanufacturers respectively under the market competition and the different strategies adopted by the government (Xi-Qiang, et.al, 2017 ). However, to clearly test their game relationship, the premise of this paper is: new products are prior to remanufactured products.

In the first period, the brand-new product is the only option available for consumers to purchase. Hence, the remanufactured is not involved during this period. Upon reaching the second period, the remanufacturer enters the market and joins the competition, thus providing consumers more options when purchasing, i.e., a brand-new product and a remanufactured product.

The remanufacturing product in the second period is restricted by the quantity of recycled used products from the first period. The decision variables of the brand-new product manufacturer and remanufacturer are the retail prices of the brand-new product and the remanufactured product. Thus, the decision objective for these two players, i.e., the manufacturer and remanufacturer, is to maximize profits. In this respect, the different types of subsidies offered by the government will influence players' behaviors in the competition and motivate consumers' behaviors when selecting the new versus the remanufactured product.The manufacturer and remanufacturer are both assumed to be economic rationalist, as they should respond to the exogenous shock rationally and maximize their own profits during the competitions.

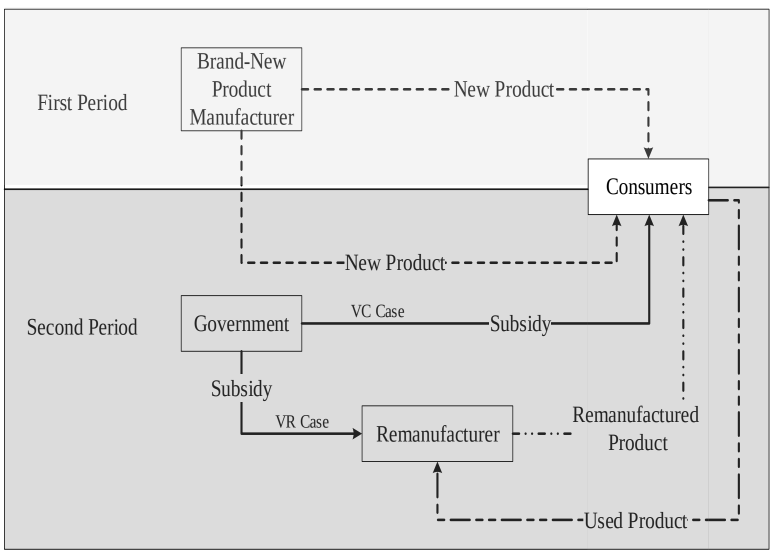


**Figure 1.** Conceptual model of the two-period game theory under different types of government subsidies

**2.2 Notations**

As shown in Figure 1, the conceptual model of the two-period game theory in different subsidies allocations is established.

While establishing the two-period models, we give clear definitions of the manufactures and remanufactures. 1)Manufacture refers to the certain industries that apply raw materials or components (homemade or purchased) to make a series of daily consumer goods through more automated machinery and equipment and production processes. The product is always brand-new and original. 2)While remanufacture shows the process of rejuvenating old machinery and equipment. It uses the old machinery and equipment as the blank, adopts special crafts and technology to make a new manufacturing on the basis of the original one, and the performance and quality of the remanufactured products are as good as the original new products. Essentially, this process belongs to recycle and recovery issues.

With the improvement of consumers' recognition of remanufactured products and the proliferation of remanufacturing technology, the original manufacturing industry has faced new conditions: increasing original equipment manufacturers and independent remanufacturers are involved in the remanufacturing field, which add to the present competition.

The following is a list of notations used in the models along with their meanings:

| | | |
|---|---|---|
| $c$ | : | manufacturing cost for single brand-new product; |
| $s$ | : | savings cost for single remanufactured product; |
| $t$ | : | preference distance for each consumer purchasing a single brand-new or remanufactured product; |
| $\tau$ | : | recovery rate of used products in the first period; |
| $e_n$ | : | impact on the environment from a single brand-new product; |
| $e_r$ | : | impact on the environment from a single remanufactured product; |
| $N$ | : | case with no subsidies; |
| $VR$ | : | case of subsidies for the remanufacturer; |
| $VC$ | : | case of subsidies for the consumer; |
| $q_{in1}$ | : | demand for brand-new product in the first period when the government chooses strategy *i*; |
| $q_{in2}$ | : | demand for brand-new product in the second period when the government chooses strategy *i*; |
| $q_{ir2}$ | : | demand for remanufactured product in the second period when the government chooses strategy *i*; |
| $p_{in1}$ | : | retail price for a single brand-new product in the first period when the government chooses strategy *i*; |
| $p_{in2}$ | : | retail price for single brand-new product in the second period when the government chooses strategy *i*; |
| $p_{ir2}$ | : | retail price for single remanufactured product in the second period when the government chooses strategy *i*; |
| $\pi_{in}$ | : | profits of brand-new product manufacturer when the government chooses strategy *i*; |
| $\pi_{ir}$ | : | profits of remanufacturer when the government chooses strategy *i*; |
| | | where $i \in \{N, VR, VC\}$ as previously mentioned; |
| $\rho$ | : | the ratio of the retail price of the brand-new product and the retail price of the remanufactured product, i.e., $0 < \rho \le 1$ |

### 2.3 Modeling Concept

This paper uses the classical demand function used by Wu (2013). This function could be described as follows: suppose that there is a preference distance for consumers when purchasing the product. We adopt x to represent the preference distance between the consumer and the remanufactured product, which is a uniform distribution of [0,1]. The distance between the consumer and the remanufactured product is 1-x as displayed in Figure 2. Thus, the utility of the consumer receiving the brand-new product is represented as $U_n = 1 - t(1-x) - p_n$, and the utility with respect to the remanufactured product is $U_r = \rho - tx - p_r$. Assuming that consumers are logical and rational, if $U_n > U_r$, the consumer prefers the brand-new product, and when $U_n < U_r$, the consumer prefers the remanufactured product.

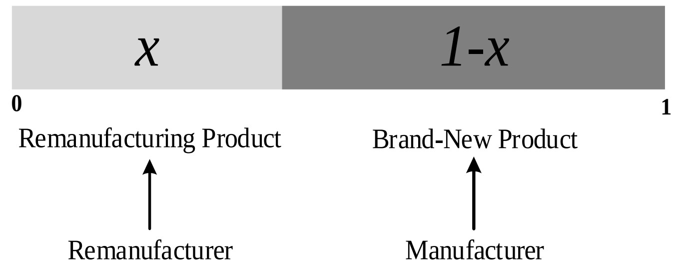


**Figure 2.** Competitive relationship between brand-new and remanufactured products

Regardless of whether the government provides a subsidies to the remanufacturer, the demand function of the brand-new product in the first period is expressed as $q_{in1} = 1 - p_{in1}$, with $i \in \{N, VR\}$.

In the second period,

$$q_{in2} = (1 + t - \rho - p_{in2} + p_{ir2}) / 2t$$

And

$$q_{ir2} = (-1 + t - \rho - p_{in2} + p_{ir2}) / 2t$$

( With $i \in \{N, VR\}$ ) represent the demand functions of the brand-new product and the remanufactured product, respectively. However, if the government subsidizes the consumer, the demand function of the brand-new product in the first period must be rewritten as

$$q_{in1} = 1 - p_{in1};$$

Then,

$$q_{VCn2} = (1 + t - \rho - v - p_{VCn2} + p_{VCr2}) / 2t,$$

And

$$q_{VCr2} = (-1 + t + \rho + v - p_{VCn2} + p_{VCr2}) / 2t$$

represent each demand function of the brand-new product and the remanufactured product, respectively, in the second period.

To simplify the derivation of the equation, let $A = -1 + \rho + 3t + s$ and $B = t\tau(1 - c)$. Accordingly, the profits of the brand-new manufacturer consist of the sales profits from the first and second periods. However, the profits of the remanufacturer are obtained only from the second period sales, and the remanufactured product supply is restricted by the quantity of used products recycled during the first period.

## 3. Model Analysis

To develop the decision-making models, this study adopts three cases: the case with no subsidies, the case with a remanufacturer subsidies and the case with a consumer subsidies (VC). The three cases are assumed to be independent and government selects one certain case during the whole period.

The models below are different from our prior papers (Qinghua, et al. 2014; Xia, et al., 2015; Qinghua, et al. 2017), as the past models has not measured and combined the two-period profits.

The case with no subsidies:

$$\max_{p_{Nn1}, p_{Nn2}} \pi_{Nn} = q_{Nn1}(p_{Nn1} - c) + q_{Nn2}(p_{Nn2} - c) \quad (1)$$

$$\max_{p_{Nr2}} \pi_{Nr} = q_{Nr2}(p_{Nr2} - c + s) \quad (2)$$

$$st. q_{Nr2} \le \tau q_{Nn1} \quad (3)$$

The case with subsidies for the remanufacturer:

$$\max_{p_{VRn1}, p_{VRn2}} \pi_{VRn} = q_{VRn1}(p_{VRn1} - c) + q_{VRn2}(p_{VRn2} - c) \quad (4)$$

$$\max_{p_{VRr2}} \pi_{VRr} = q_{VRr2}(p_{VRr2} - c + s) \quad (5)$$

$$st. q_{VRr2} \le \tau q_{VRn1} \quad (6)$$

The case with subsidies for the consumer (*VC*)

$$\max_{p_{VCn1}, p_{VCn2}} \pi_{VCn} = q_{VCn1}(p_{VCn1} - c) + q_{VCn2}(p_{VCn2} - c) \quad (7)$$

$$\max_{p_{VCr2}} \pi_{VCr} = q_{VCr2}(p_{VCr2} - c + s) \quad (8)$$

$$st. q_{VCr2} \le \tau q_{VCn1} \quad (9)$$

### 3.1 Case Without Subsidies

This subsection provides a further discussion of the relationships between the variables based on the assumption of no subsidies support. Without any government financial support, it is assumed that there merely exist pure market competitions. Their behavior is regarded as market reactions.

**Proposition 1.** Equations (1) and (2) are concave functions that are related to the decision variables PNn1, PNn2 and PNr2.

**Proof:** See appendix

The optimal solution, which is displayed in Table 1, could be obtained according to Proposition 1.

**Table1** Deduction process of the optimal solution without subsidies

| | $\tau \geq \frac{A}{3t(1-c)} = \tau_N$ | $0 < \tau \leq \tau_N$ |
|---|---|---|
| $p_{Nn1}$ | $\frac{1+c}{2}$ | $\frac{1+c}{2}$ |
| $p_{Nn2}$ | $\frac{1+3c+3t-\rho-s}{3}$ | $at+c-t\tau(1-c)$ |
| $q_{Nn1}$ | $\frac{1-c}{2}$ | $\frac{1-c}{2}$ |
| $q_{Nn2}$ | $\frac{1-3t-\rho-s}{3}$ | $\frac{2-\tau(1-c)}{3}$ |
| $q_{Nr2}$ | $\frac{A}{6t}$ | $\frac{\tau(1-c)}{2}$ |
| $\pi_{Nn}$ | $\frac{(1-c)^2}{4}+\frac{(1+3t-\rho-s)^2}{18t}$ | $\frac{(1-c)^2}{4}+\frac{t(2+\tau c-\tau)^2}{2}$ |
| $\pi_{Nr}$ | $\frac{A^2}{18t}$ | $\frac{B(A-2B)}{2t}$ |
| $p_{Nr2}$ | $\frac{A+3c-2s}{3}$ | $-1+3t+\rho+c-2t\tau(1-c)$ |

**Proposition 2.** The preference distance of purchasing a brand-new/remanufactured product has significant effects on demand, sales and sales profits. The effects on the optimal solution with different preference distances are displayed in Table 2.

**Table 2** Scenarios of effects on optimal solution with different preference distances

| | |
|---|---|
| **Scenario 1** | When $\tau \geq \tau_N$, $\frac{\partial p_{Nn2}}{\partial t} > 0$ and $\frac{\partial p_{Nr2}}{\partial t} > 0$<br>Once $\rho + s > 1$, $\frac{\partial p_{Nn2}}{\partial t} > 0$ and $\frac{\partial p_{Nr2}}{\partial t} < 0$;<br>otherwise, $\frac{\partial p_{Nn2}}{\partial t} < 0$, $\frac{\partial p_{Nr2}}{\partial t} > 0$ and $\frac{\partial \pi_{Nn}}{\partial t} = \frac{\partial \pi_{Nr}}{\partial t} > 0$ |
| **Scenario 2** | When $0 < \tau < \tau_N$, $\frac{\partial p_{Nn2}}{\partial t} > 0$ and $\frac{\partial p_{Nr2}}{\partial t} > 0$; $\frac{\partial p_{Nn2}}{\partial t} = \frac{\partial p_{Nr2}}{\partial t} = 0$;<br>$\frac{\partial \pi_{Nn}}{\partial t} > 0$ and $\frac{\partial \pi_{Nr}}{\partial t} > 0$ |

**Proof:** See appendix

Proposition 2 states that the retail price and sales profits of the brand-new/remanufactured product are directly proportional to the preference distance. While the recovery rate is higher than the margin recovery rate and the summation of the unit remanufactured product savings cost s and price discount are greater than 1, the demand for the brand-new product is directly proportional to the preference distance; otherwise, the demand for the remanufactured product is inversely proportional to the preference distance.

**Proposition 3.** The competitive relationship in the market between the brand-new product and the remanufactured product in the second period satisfies the following conditions:

Condition 1: When $\tau \geq \tau_N$ and $1-s-3t$, three scenarios must be considered as follows:

Scenario1: $0<\rho<1-s-3t$, the remanufactured product is not available in the market and $\tau=0$.

Scenario2: $1+3t-s<\rho\leq 1$, the brand-new product is not available in the market.

Scenario3: $1-s-3t<\rho<1+3t-s$, the brand-new product and the remanufactured product are both available in the market.

Condition 2: When $\tau<\tau_N$ these two products are both available in the market and all used products collected in the first period will be remanufactured by the remanufacturer.

**Proof:** See appendix

Proposition 3 states that the second period market competitive relationship between the brand-new product and remanufactured product is highly correlated with the recovery rate of used products. When the recovery rate is $\tau\geq\tau_N$ and consumers consider the ratio of the retail price to be less than $1-s-3t$, the sales of the remanufactured product is zero, which means that the remanufactured product will disappear in the second period market, as its price is highly valued. However, when the consumers perceive the ratio of the retail price to be greater than $1+3t-s$, they will no longer choose the brand-new product. When the ratio of retail price remains in the condition $1-s-3t<\rho<1+3t-s$, the brand-new product and the remanufactured product are both available in the market.

When the recovery rate of the used product in the first period is less than $\tau_N$, the demand for the remanufactured product is greater than the collection of the used products, which means that all used products will be remanufactured. However, since the quantity of the remanufactured product is constrained by the quantity of the used products, some consumers will select the brand-new product as a substitute.

### 3.2 Case with Subsidies for Remanufacturers

Assuming the government subsidizes the remanufacturer, the optimal solution is shown as Table 3.

**Table 3** Optimal solutions when the strategy is VR

| | $\tau\geq\dfrac{A+v}{3t(1-c)}=\tau_{VR}$ | $0<\tau\leq\tau_{VR}$ |
|---|---|---|
| $p_{VRn1}$ | $\dfrac{1+c}{2}$ | $\dfrac{1+c}{2}$ |
| $p_{VRn2}$ | $\dfrac{1+3c+3t-\rho-s-v}{3}$ | $2t+c-B$ |
| $p_{VRr2}$ | $\dfrac{A+3c-2s-2v}{3}$ | $-1+3t+\rho+c-2t\tau(1-c)$ |
| $q_{VRn1}$ | $\dfrac{1-c}{2}$ | $\dfrac{1-c}{2}$ |
| $q_{VRn2}$ | $\dfrac{1-3t-\rho-s-v}{3}$ | $\dfrac{2-\tau(1-c)}{3}$ |
| $q_{VRr2}$ | $\dfrac{A+v}{6t}$ | $\dfrac{B}{2t}$ |
| $\pi_{VRn}$ | $\dfrac{(1-c)^2}{4}+\dfrac{(1+3t-\rho-s-v)^2}{18t}$ | $\dfrac{(1-c)^2}{4}+\dfrac{t(2+\tau c-\tau)^2}{2}$ |
| $\pi_{VRr}$ | $\dfrac{(A+v)^2}{18t}$ | $\dfrac{B(A-2B)}{2t}$ |

### 3.3 Case with Subsidies for Consumers

Assuming the government subsidizes the consumer to promote the selection of the remanufactured product, the optimal solution for the government is shown in Table 4.

**Table 4** Optimal solutions when the strategy is VC

| | $\tau \geq \frac{A-v}{3t(1-c)} = \tau_{VC}$ | $0 < \tau \leq \tau_{VC}$ |
|---|---|---|
| $p_{VCn1}$ | $\frac{1+c}{2}$ | $\frac{1+c}{2}$ |
| $p_{VCn2}$ | $\frac{1+3c+3t-\rho-s-2v}{3}$ | $2t+c-B$ |
| $p_{VCr2}$ | $\frac{-1+3c+3t+\rho-2s-v}{3}$ | $-1+3t+\rho+c-2t\tau(1-c)$ |
| $q_{VCn1}$ | $\frac{1-c}{2}$ | $\frac{1-c}{2}$ |
| $q_{VCn2}$ | $\frac{1-3t-\rho-s-2v}{3}$ | $\frac{2-\tau(1-c)}{3}$ |
| $q_{VCr2}$ | $\frac{A+2v}{6t}$ | $\frac{B}{2t}$ |
| $\pi_{VCn}$ | $\frac{(1-c)^2}{4}+\frac{(1+3t-\rho-s-2v)^2}{18t}$ | $\frac{(1-c)^2}{4}+\frac{t(2+\tau c-\tau)^2}{2}$ |
| $\pi_{VCr}$ | $\frac{(A-v)(A+2v)}{18t}$ | $\frac{B(A+v-2B)}{2t}$ |

**Proposition 4.** The retail price and the demand are affected by the government subsidies in the second period.

Scenario 1: The government subsidizes the remanufacturer:

When $\tau \geq \tau_{VR}$, $\frac{\partial p_{VRn2}}{\partial v} < 0$, $\frac{\partial p_{VRr2}}{\partial v} < 0$, $\frac{\partial q_{VRn2}}{\partial v} < 0$, $\frac{\partial q_{VRr2}}{\partial v} < 0$

when $\tau < \tau_{VR}$,

$$\frac{\partial p_{VRn2}}{\partial v} = \frac{\partial p_{VRr2}}{\partial v} = \frac{\partial q_{VRn2}}{\partial v} = \frac{\partial q_{VRr2}}{\partial v} = 0$$

Scenario 2: The government subsidizes the consumer rather than the remanufacturer:

When $\tau \geq \tau_{VC}$, $\frac{\partial p_{VRn2}}{\partial v} < 0$, $\frac{\partial p_{VRr2}}{\partial v} < 0$, $\frac{\partial q_{VRn2}}{\partial v} < 0$, $\frac{\partial q_{VRr2}}{\partial v} > 0$

when $\tau < \tau_{VC}$,

$$\frac{\partial p_{VRn2}}{\partial v} = \frac{\partial q_{VRr2}}{\partial v} = \frac{\partial q_{VRn2}}{\partial v} 0, \frac{\partial p_{VRr2}}{\partial v} > 0$$

**Proof:** See appendix

Proposition 4 states that government subsidies enable the reduction of the retail price of both the brand-new product and the remanufactured product when $\tau \geq \tau_{VR}$ or $\tau \geq \tau_{VC}$. Government subsidies lowers the retail price of a brand-new product because the new product will lose its price advantage when the price of the remanufactured product is reduced. Consumers may turn to remanufactured product and show less demand of new ones.

For the new product to maintain a price advantage, the manufacturer reduces the retail price to attract consumers and compete with the remanufactured product. If $\tau < \tau_{VR}$, then the demand for the remanufactured product is greater than the supply of the used product, and the government subsidies do not impact the retail price or demand. However, the second period demand for the remanufactured product will exceed the quantity of the used product in the first period when $\tau < \tau_{VC}$

and the government subsidizes the consumer, which causes the remanufacturer to increase the retail price to obtain greater profit. In other words, the retail price of the remanufactured product is increased based on the amount of the government subsidies. In response to the sustainability of Green Economy, government ought to allocate proper amounts of subsidies to support the competitiveness of remanufactures in the market, as they are beneficial to circular economy. Proposition 6 in the latter part would consider this question.

**Proposition 5.** The impact of government subsidies on retail prices and sales of brand-new and remanufactured products is shown in Table 5.

**Table 5** Retail prices and sales of brand-new and remanufactured products under different government subsidies

| Conditions | Results |
|---|---|
| $\tau \geq \tau_{VR}$ | (a) $p_{VCn2} < p_{VRn2} < p_{Nn2}$, $p_{VRr2} < p_{VCr2} < p_{Nr2}$<br>(b) $q_{VCn2} < q_{VRn2} < q_{Nn2}$, $q_{Nr2} < q_{VRr2} < q_{VCr2}$ |
| $\tau_N \leq \tau \leq \tau_{VR}$ | (a) $p_{VCn2} < p_{VRn2} < p_{Nn2}$,<br>(b)if $v > 2(3B-A)$, $p_{VCr2} < p_{VRr2} < p_{Nr2}$,<br>otherwise, $p_{VRr2} < p_{VCr2} < p_{Nr2}$<br>(c) $q_{VCn2} < q_{VRn2} < q_{Nn2}$, $q_{Nr2} < q_{VRr2} < q_{VCr2}$ |
| $\tau_{VC} \leq \tau \leq \tau_N$ | (a) $p_{VCn2} < p_{VRn2}=p_{Nn2}$, $p_{VCr2} < p_{VRr2}=p_{Nr2}$<br>(b) $q_{VCn2} < q_{VRn2}=q_{Nn2}$, $q_{Nr2}=q_{VRr2} < q_{VCr2}$ |
| $\tau < \tau_{VC}$ | (a) $p_{VCn2}=p_{VRn2}=p_{Nn2}$, $p_{VCr2} > p_{VRr2}=p_{Nr2}$<br>(b) $q_{VCn2} < q_{VRn2}=q_{Nn2}$, $q_{Nr2}=q_{VRr2}=q_{VCr2}$ |

**Proof:** See appendix

Proposition 5 states that if the used product recovery rate is $\tau \geq \tau_{VR}$, then the optimal decision of a brand-new product in the second period is $N > VR > VC$ with respect to the retail price, sales and profits. For the remanufactured product, if $N > VC > VR$ is the optimal retail price order, then the optimal sales rank is $VC > VR > N$.

In addition, if the recovery rate is $\tau_N \leq \tau \leq \tau_{VR}$, then the retail price and sales exhibit the same changes in the second period. Once the amount of the government subsidies v is greater than $2(3B-A)$, then the retail price of the remanufactured product is $N > VR > VC$, and if $v < 2(3B-A)$, then the retail price order becomes $N > VC > VR$. Subsequently, the second period retail price of a brand-new product is a special case in condition 2, where the recovery rate is $\tau_{VR} \leq \tau \leq \tau_N$.

Assuming that $\tau < \tau_{VC}$, the supply of the remanufactured product in the second period is restricted by the quantity of recycled used products in the first period. Thus, the retail price and the sales of the remanufactured product are subject to the same situation as that in the case of N or VR. However, remanufacturers attempt to increase the retail price of the remanufactured product by transferring part of the government subsidies as in the case of VC. Although the retail price is higher than that of the N and VR cases, the sales reflect the same situation as these two cases because the demand for the remanufactured product is greater than the supply.

Proposition 6. How government subsidies affect the sales profits of brand-new and remanufactured products is shown in Table 6.

**Table 6** Sales profits of brand-new and remanufactured products under different Government subsidies

| Conditions | Results |
|---|---|
| $\tau \geq \tau_{VR}$ | $\pi_{VCn} < \pi_{VRn} < \pi_{Nn}$,<br>when $v > 2$, $\pi_{VCr} < \pi_{Nr} < \pi_{VRr}$, |

| | |
|---|---|
| | Otherwise, $\pi_{Nr} < \pi_{VCr} < \pi_{VRr}$ |
| $\tau_N \le \tau < \tau_{VR}$ | $\pi_{VCn2} < \pi_{VRn2} < \pi_{Nn2}$, when $0 \le v < \frac{6B-A}{2}$, $\pi_{VRr} < \pi_{Nr} < \pi_{VCr}$, Otherwise, $\pi_{Nr} < \pi_{VCr} < \pi_{VRr}$ |
| $\tau_{VC} \le \tau < \tau_N$ | $\pi_{VCn} < \pi_{VRn} = \pi_{Nn}$, when $0 \le v < \frac{6B-A}{2}$, $\pi_{VRr} = \pi_{Nr} < \pi_{VCr}$, Otherwise, $\pi_{VCr} < \pi_{VRr} = \pi_{Nr}$ |
| $\tau < \tau_{VC}$ | $\pi_{VCn} = \pi_{VRn} = \pi_{Nn}$, $\pi_{Nr} = \pi_{VRr} < \pi_{VCr}$ |

**Proof:** See appendix

**Proposition 6** states that the sales profits of remanufacturer are affected by the amount of the government subsidies. When the amount of government subsidies reaches $v > A/2$ ,the remanufacturer realizes the smallest sales profit. Furthermore, the reduction range of the retail price for a brand-new product is greater than that for are manufactured product (vicious price-competition by means of cut-throat strategies may occur). Thus, the remanufactured product does not have a competitive price advantage in the VC case. In particular, the unit sales profit of the remanufactured product is reduced if the amount of government subsidies continues to increase. Accordingly, the subsidies amount must be less than $A/2$ in the VC case for the sales of the remanufactured product to reach the maximum point and for the sales profit to exceed that of case N. Once the subsidies amount exceeds $A/2$ , although the sales of the remanufactured product can increase, the sales profit will be less than that of case N because of the effect of reducing the price for the brand-new product. These economic behavior ought to be adjusted to promote normal competition.

**Proposition 7.** Different government strategies affect the recovery rate of the used product differently, $\tau_{VC} < \tau_N < \tau_{VR}$.

**Proof:** See appendix

Proposition 7 states that the margin recovery rate enables the achievement of the maximum point in the VR case because the remanufacturer intends to aggressively promote the recovery rate of the used product to acquire additional government subsidies. Accordingly, the government can exploit this point to encourage the remanufacturer to achieve the recovery rate $\tau_{VR}$ and thus attain the subsidies target.

Once the government subsidizes consumers (VC), the margin recovery rate remains at the minimum point, although it will still encourage consumers to select the remanufactured product and promote consumer recycling behaviors. This effect of VC on remanufactured product is not precise and accurate, as the demand of brand-new product is also increasing. Consequently, the government must encourage remanufacturers to maintain the highest recovery rate in the case of VC. Remanufacturers can even retain the recovery rate as in the N case.

Based on the studies of Geylani et al. (2007) and Agrawal et al. (2012), the environmental impact $e_i$ can be obtained in the N, VR and VC cases as follows:

$$e_N = e_n(q_{Nn1} + q_{Nn2}) + e_r q_{Nr2}$$

$$e_{VR} = e_n(q_{VRn1} + q_{VRn2}) + e_r q_{VRr2}$$

$$e_{VC} = e_n(q_{VCn1} + q_{VCn2}) + e_r q_{VCr2}$$

Proposition 8. Different government subsidies strategies will affect the environment.

a) If $\tau \ge \tau_N$, $e_N > e_{VR} > e_{VC}$

b) If $\tau_{VC} \le \tau < \tau_N$, $e_N = e_{VR} > e_{VC}$

c) If $\tau < \tau_{VC}$, $e_N = e_{VR} = e_{VC}$

**Proof:** See appendix

Having considered the economic consequences in different cases, the effect of government subsidies on ecology is to be examined. The recovery rate is supposed to be related to Green Economy. Proposition 8 indicates that if the recovery rate $\tau$ is greater than $\tau_{VR}$ in the VC case, then the environmental impact is less than that of any of the other situations because the subsidies result in an increase in the sales of the remanufactured product, which causes a decrease in the demand for the brand-new product.

Conversely, case N generates an enormous environmental impact. In the VR case, if $\tau_N \leq \tau < \tau_{VR}$, the demand for the remanufactured product is restricted by the quantity of the used product in the first period. Hence, the remanufactured product is unable to satisfy the market demand. Therefore, consumers choose to select the brand-new product as a substitute, thereby generating a tremendous impact on the environment. When $\tau_{VC} \leq \tau < \tau_N$, then the supply of the remanufactured product in the second period is still restricted by the recycled quantity of the used product in the first period, although the demand for the brand-new product and the remanufactured product face the same situation as in the N and VR cases, which then results in the same degree of environmental impact. In other words, N, VC, and VR have the same environmental impact if $\tau < \tau_{VC}$.

## 4. Conclusions and Discussions

### 4.1 Conclusions

Considering the importance of social benefits, Chinese government gradually emphasize more subsidies on environmental sustainability, and remanufacturing industry is rising. The game between manufacturing and remanufacturing industries in the market has formed. Based on the given condition, this study adopts the Stackelberg two-period game theory based on the Chinese government proposed policy "Exchange your old product for a manufactured one" to investigate the effects under three cases of different types of subsidies. The effects are investigated by developing mathematical models that incorporate government subsidies, retail prices, preference distances, market demand and environmental impacts. These mathematical derivations support policy making and can be used to examine dynamic changes. Also, the results of derivations offer economic guide to regulators and entrepreneurship about price competitions and subsidy utility.

Firstly, government subsidies play an important role in reducing the retail price of new and remanufactured products both in the case of subsidizing the remanufacturer (VR) and the case of subsidizing consumer (VC). If $\tau < \tau_{VR}$, then the demand for the remanufactured product in the second period is restricted by the recovery rate of used products in the first period, and government subsidies to the remanufacturer will not affect the demand and retail prices of brand-new and remanufactured products. When the government subsidizes consumers and $\tau < \tau_{VC}$, the remanufacturer strives to acquire more profits by increasing the retail price and shifting the government subsidies. In other words, the retail price of the remanufactured product depends on the quantity of subsidies. Although government subsidies are conducive to promoting the sale of remanufactured products, remanufacturers also encounter sales profit margin reductions. Therefore, government subsidy is restricted by recovery rate in the first period and ought to balance the conflicts among these objectives. The enterprises could not ignore the potential risks and losses made by their actions.

Secondly, government subsidies cannot exceed the value of A/2. Once the subsidies have exceeded this value, they will generate vicious competition between the manufacturers of the new and remanufactured products, and the remanufacturer can only receive the minimum sales profit. If the government subsidizes consumers, then the brand-new product will cause a greater decrease in the retail price. In other words, the remanufactured product will lose its price advantage. Although the government promotes the sales of remanufactured products by subsidizing consumers, the sales profits of the remanufacturer still fail to increase because of the government subsidies that reduces the retail price. Therefore, the impact of government subsidy is not always positive, and rules and regulations require modifications and improvements.

Thirdly, the findings indicate that adopting different strategies will generate different impacts on the environment. If the recovery rate of the remanufacturers is greater than the government's margin recovery rate without government subsidies, then the case with subsidies for consumers (VC) will present a significant reduction in the impacts on the environment. Once the recovery rate of the remanufacturer is located between the margin recovery rates of the two cases, i.e., the case with subsidies for the remanufacturer (VR) and the case without subsidies (N), ($\tau_{VC} \leq \tau < \tau_N$), then the government subsidies to consumers can minimize the environmental impact; otherwise, the same impact is observed in both cases. In addition, these three cases have the same impact on the environment when the recovery rate of the remanufacturer is less than the margin recovery rate in the case with subsidies for consumers. The Green Economy is highly related to the recovery rate caused by the two period mentioned above.

**4.2 Discussions**

Green Economy is a new economic form, which is market-oriented, based on the traditional industrial economy and aiming at the harmony between economy and environment. It is a kind of development state that the industrial economy produces and shows to meet the need of human environmental protection and health. Remanufacturing industry could reflect the purpose of Green Economy to a large extent. The rise of this new industry and government financial support, however, may even produce adverse effect including price-competitions and negative marketing strategies.

Then this paper has illustrated the significant effect of government subsidies on manufacturing and remanufacturing industries by formula derivation, which has seen the mathematical logistics. Although Green Economy has been proposed worldwide, and governments in different areas have increased subsidies towards environmental issues, the efficiency and effectiveness still have to be adjusted to a new level. The adjustments require clear position of objectives (who has the priority to receive subsidy) and accurate measurement of the investment and changes. Trade-offs between surplus on remanufactures (e.g., boosting market demand) and potential losses (e.g., vicious competition with brand-new product & decline in profits) should be considered at the same time. Overall, the net surplus, especially from the perspective of Green Economy, requires to be positive.

This study develops an optimal and dynamic method of managing government subsidies in the event of a social surplus. Our past papers just analyze their interactions limited in one period, and this paper introduces the entrance of remanufacturers in the next period. Therefore, the results enable us to offer scientific support to governments when developing policies about subsidies to augment the development of the remanufacturing industry. Governments ought to balance the benefits between social responsibility and economy; the imposed recovery rate should also be proper.

Although this study explores government subsidies, preference distances, demand, sales profits and environmental impacts in the development of an extensive model to investigate the effects under different scenarios, several limitations also exists. For example, in the case of subsidies how does the government confirm the group to receive the subsidies? The precise amount of the subsidies was not addressed, and it should be discussed further in future research. To simplify the problem of government subsidies and optimal solutions, we did not consider some affected variables and recommend that they can be included in a future study.

**APPENDIX**

**Proof of Proposition 1**

Substituting

$$q_{Nn1} = 1 - p_{Nn1},\ q_{Nn2} = \frac{1+t-\rho-p_{Nn2}+p_{Nr2}}{2t}$$

into Eq. (1), we obtain

$$\max_{p_{Nn1},p_{Nn2}} \pi_{Nn} = (p_{Nn1}-c)(1-p_{Nn1}) + (p_{Nn2}-c)\frac{1+t-\rho-p_{Nn2}+p_{Nr2}}{2t} \tag{10}$$

Eq. (10) yields first and second order partial derivatives and can obtain

$$\frac{\partial \pi_{Nn}}{\partial p_{Nn1}} = 1 - 2p_{Nn1} + c,$$

$$\frac{\partial \pi_{Nn}}{\partial p_{Nn2}} = \frac{1+t+c-\rho-2p_{Nn2}+p_{Nr2}}{2t},$$

$$\frac{\partial^2 \pi_{Nn}}{\partial {p_{Nn1}}^2} = -2,\ \frac{\partial^2 \pi_{Nn}}{\partial {p_{Nn2}}^2} = -\frac{1}{t},$$

$$\frac{\partial^2 \pi_{Nn}}{\partial p_{Nn1}\partial p_{Nn2}} = \frac{\partial^2 \pi_{Nn}}{\partial p_{Nn2}\partial p_{Nn1}} = 0.$$

Thus,

$$J = \begin{bmatrix} \frac{\partial^2 \pi_{Nn}}{\partial {p_{Nn1}}^2} & \frac{\partial^2 \pi_{Nn}}{\partial p_{Nn1}\partial p_{Nn2}} \\ \frac{\partial^2 \pi_{Nn}}{\partial p_{Nn2}\partial p_{Nn1}} & \frac{\partial^2 \pi_{Nn}}{\partial {p_{Nn2}}^2} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -\frac{1}{t} \end{bmatrix} = \frac{2}{t} > 0$$

and $\frac{\partial^2 \pi_{Nn}}{\partial {p_{Nn1}}^2} = -2 < 0$. This confirms that $p_{Nn1}$ and $p_{Nn2}$ are concave functions of Eq. (1).

Subsequently, inserting $q_{Nn1} = 1 - p_{Nn1}$ and

$q_{Nr2}=\frac{-1+t+\rho+p_{Nn2}-p_{Nr2}}{2t}$ into Eq. (2) and considering Eq. (3) as a constraint condition, Eq. (2) is rewritten as a non-constraint Lagrange function ,

$$\max_{p_{Nr2}}\pi_{Nr}=\frac{(p_{Nr2}-c+s)(-1+t+\rho+p_{Nn2}-p_{Nr2})}{2t}$$
$$+L[\frac{-1+t+\rho+p_{Nn2}-p_{Nr2}}{2t}-\tau(1-p_{Nn1})]$$

and $L>0$ is a Lagrange multiplier. Accordingly, $\pi_{Nr}$ is related to the second order function of $p_{Nr2}$, and $\frac{\partial^2\pi_{Nr}}{\partial p_{Nr2}{}^2}=-\frac{1}{2t}<0$.Thus, Eq. (2) is a concave function that relates to $p_{Nr2}$. Similarly, these derivations can be used to show that Eq. (1) is a concave function with respect to $q_{Nn1}$ and $q_{Nn2}$ and Eq. (2) is a concave function for $q_{Nn2}$ as well.

**Proof of Proposition 2**

a) when $\tau\geq\tau_N$, we know

$$\frac{\partial p_{Nn2}}{\partial t}=\frac{\partial p_{Nr2}}{\partial t}=1>0.$$

Considering $\frac{\partial q_{Nn2}}{\partial t}=\frac{-1+\rho+s}{6t^2}$

and

$$\frac{\partial q_{Nr2}}{\partial t}=\frac{1-\rho-s}{6t^2},$$

we find that when

$\rho+s>1$, $\frac{\partial q_{Nn2}}{\partial t}>0$ and $\frac{\partial q_{Nr2}}{\partial t}<0$;

otherwise,

$$\frac{\partial q_{Nn2}}{\partial t}<0,\frac{\partial p_{Nr2}}{\partial t}>0$$

And

$$\frac{\partial\pi_{Nn}}{\partial t}=\frac{\partial\pi_{Nr}}{\partial t}=\frac{(1+3t-\rho-s)A}{18t^2}>0.$$

b) when $\tau<\tau_N$,

we know $\frac{\partial p_{Nn2}}{\partial t}=2-\tau(1-c)>0$

and $\frac{\partial p_{Nr2}}{\partial t}=3-2\tau(1-c)>0$;

Since $t$ is not involved in $q_{Nn2}$ and $q_{Nr2}$, we get

$$\frac{\partial q_{Nn2}}{\partial t}=\frac{\partial q_{Nr2}}{\partial t}=0;$$

meanwhile,

$$\frac{\partial\pi_{Nn}}{\partial t}=\frac{(2+\tau c-\tau)^2}{2}>0$$

and

$$\frac{\partial\pi_{Nr}}{\partial t}=\frac{\tau(1-c)(3+2\tau c-2\tau)}{2}>0.$$

**Proof of Proposition 3**

a) Suppose $\tau\geq\tau_N$. A brand-new product in the second period market is expressed as

$$q^*_{Nn2}=\frac{1+3t-\rho-s}{6t}>0,$$

which means that $\rho<1+3t-s$. If remanufacturing products are in the market, then this situation presents as $q^*_{Nr2}=\frac{A}{6t}>0$, which means that $1-s-3t<\rho$. Consequently, remanufacturing products will disappear in the market when $0<\rho<1-s-3t$. Once $0<\rho<1-s-3t$, a brand-new product will disappear in the market. If $1-s-3t<\rho<1+3t-s$, the new product and the remanufactured product can coexist simultaneously in the market.

b) When $\tau<\tau_N$,

$$q^*_{NRn2}=\frac{2-\tau(1-c)}{2}>\frac{1+c}{2}>0,$$

$q^*_{NRr2}=\frac{\tau(1-c)}{2}>0$. This indicates that the brand-new product and the remanufactured product coexist in the market.

**Proof of Proposition 4**

a) When $\tau\geq\tau_{VR}$,

$$\frac{\partial p_{VRn2}}{\partial v}=\frac{-1}{3}<0,\frac{\partial p_{VRr2}}{\partial v}=\frac{-2}{3}<0,$$

$$\frac{\partial q_{VRn2}}{\partial v} = \frac{-1}{6t} < 0, \frac{\partial q_{VRr2}}{\partial v} = \frac{-1}{6t} > 0.$$

However, When $\tau < \tau_{VR}$, $p_{VRn2}$, $p_{VRr2}$, $q_{VRn2}$ and $q_{VRr2}$ have no relations with $v$, i.e., the conclusions are correct.

b) The proof of b) is similar to a)□

**Proof of Proposition 5**

a) When $\tau \geq \tau_{VR}$, we obtain (a) and (b) as follows:

From

$$p^*_{Nn2} - p^*_{VRn2} = p^*_{VRn2} - p^*_{VCn2}$$

$$= p^*_{Nr2} - p^*_{VCr2} = p^*_{VCr2} - p^*_{VRr2} = \frac{v}{3} > 0,$$

we find that $p^*_{VCn2} < p^*_{VRn2} < p^*_{Nn2}$ and $p^*_{VRr2} < p^*_{VCr2} < p^*_{Nr2}$. Thus, the conclusion is confirmed.

Using the same derivation as above, the proposition is confirmed.

b) Because conditions 2 to 4 have the same derivations as condition 1, the proofs are omitted to avoid redundancy.

c) Condition 5 is confirmed by comparing the retail price and the demand for the brand-new product in the first period under different cases.

**Proof of Proposition 6,7**

The proof of proposition 6 and 7 is proven using the same derivations as those in proposition 5.

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