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Improving the Performance of EWMA mean chart with use of Two Auxiliary Variables

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Abstract
Control chart is magnificent monitoring technique applied in tracing undesired variation in the manufacturing and services processes. Quality practitioners are always in search of cost-effective monitoring model to produce high quality products. Auxiliary information-based procedures provide unbiased and more efficient estimate about the process mean. The exponentially weighted moving average (EWMA) scheme based on regression estimator using two auxiliary variables is proposed for efficient monitoring of the process mean in this study. The performance of the proposed design is reported under the collinear and non-collinear cases of the two auxiliary variables. The sensitivity of the proposed chart is compared with classical EWMA, single use of auxiliary EWMA, classical homogeneously weighted moving average (HWMA) and single auxiliary information based HWMA control charts. The comparison reveals the superiority of the proposed chart over its competitors. An application is also included to support the theory.
Keywords: Auxiliary characteristics; Average run length; Efficient control chart; Multicollinearity; Regression estimator;

1. Introduction

In the manufacturing industry, statistical process control (SPC) is widely used to uphold stability of the process by reducing the amount of variation in it. Common causes and special causes of variation occur in the output of any ongoing process. Statistically, the process is in-control (IC) if natural changes occur in the process and the un-natural changes refer the process out-of-control (OOC) (cf. Montgomery (2012)). Practitioners urge to improve the quality of process output by timely detection of un-natural variation. To trace un-natural changes in the process parameters, control charts are commonly applied. Control charts are divided into memory-type and memoryless according to their designed structures. The charting schemes based on Shewhart (1924) mechanism are known as memoryless as these monitor the process based on the most recent observation. On the other hand, charting designs which incorporate lag information along with the most recent information such as; cumulative sum (CUSUM) by page (1954) and exponentially weighted moving average (EWMA) by Roberts (1959) are referred as memory-type control charts.

For measuring relative efficiency of the charts average run length (ARL) is usually used as performance measure, which is the expected number of run lengths. A chart, possessing small value of ARL when it is working under OOC state, is nominated as superior. To enhance the detection ability of control charts, several new techniques are proposed by researchers. In sampling survey and estimation techniques concept of the auxiliary information is frequently used as these produces unbiased and efficient estimators of the process parameter(s). It is well known property that regression estimation is the most efficient way of using the auxiliary information which can be expressed as information accessible at the estimation stage other than that sampled information is known as auxiliary information (cf. Abbas et al. (2014)). In SPC, auxiliary information is commonly used to enhance the detection abilities of the control charts. In regression-type control

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charts monitoring characteristic depends on supporting characteristic (cf. Mandel (1969) and Zhang (1985)). The proper auxiliary based control charts were initiated by Riaz (2008a) and Riaz (2008b) for monitoring the process location and variability respectively. Abbas et al. (2014) proposed single auxiliary information EWMA (AEWMA) chart to enhance the detection ability of EWMA chart. After that many researchers developed control chart using auxiliary information few are; Riaz A, (2019), Haq and Bibi (2019), Abbas et al. (2020). JH Chen, and SL Lu, (2020) and Amir et al. (2020).

To improve shift detection ability in coefficient of variation an auxiliary information-based chart was proposed by Abbasi (2020). For monitored the location parameter Mixed EWMA Dual-CUSUM chart was proposed by Abbas et al. (2018).

All the articles cited above are auxiliary information-based control charts using single supporting variable, however in practical situation more than two variables effect the performance of the study variable. In this article, EWMA chart is being constructed when the study characteristic depends on two auxiliary characteristics (TAEWMA, named hereafter). The performance of the proposed TAEWMA chart is measured in the presence and absence of multicollinearity between the two auxiliary variables. Rest of the study is organized as: Section 2 describes structures of few existing charts. Development of the proposed chart provided in section 3. In Sections 4 the RL evaluation is performed. Section 5 consists of comparison and application is presented in Section 6. The article ends with conclusion and recommendations.

2. Designs of some existing chart,

This section briefly describes some existing charts i.e. classical EWMA, classical HWMA, AEWMA and AHWMA charts.

2.1. Design Structure of classical EWMA chart.

The EWMA chart to monitor process mean was designed by Roberts (1959). Suppose Y represents a random variable whose values are taken from normal distribution with mean μ_0 and variance σ_Y^2respectively. For monitoring the process mean EWMA statistic introduced by (cf. Roberts, 1959) can be defined as:

$$Z_i = \lambda \overline{Y}_i + (1-\lambda)Z_{i-1}, \quad (1)$$

Where λ is known smoothing constant and it is chosen between $0 < \lambda \leq 1$. Here i is the sample number and $\overline{Y}_i = \frac{\sum_{i=1}^{n} Y_i}{n}$, is the average of ithsample. In this study, without loss of generality size of the sample is taken equal to one. In equation one Z_i shows EWMA plotting statistic and quantity Z_{i-1} stands for EWMA plotting statistic at time (i-1). when the process is IC, the $E(Z_i) = \mu_0$ and $V(Z_i) = \sigma_{\overline{Y}}^2(\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i}))$.

The upper control limit (UCL), lower control limit (LCL) and central line (CL) of the EWMA chart are given as:

$$\left.\begin{aligned} LCL &= \mu_0 - L\sigma_{\overline{Y}}\sqrt{(\tfrac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i}))}, \\ CL &= \mu_0, \\ UCL &= \mu_0 + L\sigma_{\overline{Y}}\sqrt{(\tfrac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i}))}, \end{aligned}\right\}. \quad (2)$$

Here, $\sigma_{\overline{Y}} = \frac{\sigma_Y}{\sqrt{n}}$ and L represents as control limits coefficient and it depends on the predefined ARL_0 and the choices of weighting constant λ. The classical EWMA chart detects OOC state if at any sample number $Z_i$goes outsides the control limits given in equation 2.

2.2. Design Structure of Auxiliary based EWMA chart (AEWMA).

The AEWMA charting scheme was proposed by Abbas et al. (2014) for efficient monitoring of the process mean. The correlation between two variables is denoted by ρ_{YX}, where X_i is an auxiliary variable is correlated with variable of interest Y_i. For bivariate normal distribution Y and X can be expressed as $(Y, X) \sim N_2(\mu_{Y,}\mu_X, \sigma_Y^2, \sigma_X^2, \rho_{YX})$. Here $N_2$is assumed for bivariate normal distribution. Regression estimator for papulation mean $\mu_0$following Cochran (1977) can be written as

$$A_Y = \overline{Y} + b_{YX}(\mu_X - \overline{X}), \quad (3)$$

In equation (3), b_{YX} measures change in Y due to one unit change in X. where $b_{YX} = \rho_{YX}\left(\frac{\sigma_Y}{\sigma_X}\right)$ and mean and variance of A_Y respectively are given below.

$$E(A_Y) = \mu_0, V(A_Y) = \sigma_A^2 = \frac{\sigma_Y^2}{n}(1-\rho^2{}_{YX}) = \frac{\sigma_Y^2 - b_{YX}^2\sigma_X^2}{n} \quad (4)$$

Equation (4) suggested that $A_Y$is unbiased estimator of $\mu_0$and variance is $V(A_Y) < V(\overline{Y})$ for $\rho^2{}_{YX} > 0$. The AEWMA statistic based on regression estimator in defined as

$$M_i = \lambda A_{Yi} + (1-(1-\lambda))M_{i-1} \quad (5)$$

Here λ is representing the smoothing constant for AEWMA , A_{Yi} is the value of statistic A_Y for the i^{th} sample. M_{i-1} denotes as prior information and it takes initial values of M_0which is equal to target mean μ_0. The control limits of AEWMA are given as,

$$\left.\begin{aligned} LCL_i &= \mu_0 - L\sigma_{\overline{Y}}\sqrt{(1-\rho^2{}_{YX})(\tfrac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i}))},\\ CL &= \mu_0,\\ UCL_i &= \mu_0 + L\sigma_{\overline{Y}}\sqrt{(1-\rho^2{}_{YX})(\tfrac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i}))}, \end{aligned}\right\}. \quad (6)$$

The AEWMA chart alarms if at any sample point the plotting statistic goes outside the control limits.

2.3. Design Structure of classical homogeneous weighted moving average (HWMA) control chart.

The HWMA charting scheme is a memory-type chart and was proposed by Abbas (2018) to monitor the process mean. The mathematical formulation of HWMA statistic is,

$$H_i = \lambda\overline{Y}_i + (1-\lambda)\overline{Y}_{i-1} \quad (7)$$

Where $\overline{Y}_i$represent the sample average of i^{th} sample and $\lambda$is the sensitivity parameter of HWMA chart. Sensitivity parameter always takes values between zero and one. In equation (7) $\overline{Y}_{(i-1)} = \frac{\sum_{k=1}^{i-1} Y_k}{i-1}$, represents the mean of the means of previous $(i-1)$ samples. $\overline{Y}_0$is a target mean which is equal to μ_0. In equation (7) can be expressed as.

$$H_i = \lambda\overline{Y}_i + \left[\left(\frac{1-\lambda}{i-1}\right)\overline{Y}_{i-1} + \left(\frac{1-\lambda}{i-1}\right)\overline{Y}_{i-2} + \ldots + \left(\frac{1-\lambda}{i-1}\right)\overline{Y}_1\right]. \quad (8)$$

In above equation λ represents the weight of current sample and remaining $(1-\lambda)$ weights is the homogeneously distributed for all the previous sample information.

$$\left.\begin{aligned} LCL_i &= \begin{cases} \mu_0 - C\sqrt{\frac{\lambda^2\sigma_Y^2}{n}}, & \text{if i} = 1\\ \mu_0 - C\sqrt{\frac{\lambda^2\sigma_Y^2}{n} + (1-\lambda)^2\frac{\sigma_Y^2}{n(i-1)}}, & \text{if i} > 1 \end{cases}\\ CL &= \mu_0,\\ UCL_i &= \begin{cases} \mu_0 + C\sqrt{\frac{\lambda^2\sigma_Y^2}{n}}, & \text{if i} = 1\\ \mu_0 + C\sqrt{\frac{\lambda^2\sigma_Y^2}{n} + (1-\lambda)^2\frac{\sigma_Y^2}{n(i-1)}}, & \text{if i} > 1 \end{cases} \end{aligned}\right\} \quad (9)$$

Where C represent the width of control limits and it is based on ARL_0. If plotting statistic of HWMA control chart goes beyond the control limits, process is said to be OOC.

2.4 Design Structure of auxiliary homogeneous weighted moving average (AHWMA) control chart.

The AHWMA chart for tracing shift in process mean is recently proposed by Nurudeen et al. (2019). The regression estimator for monitoring mean of the process is defined as;

$$S_i = \overline{Y}_i + b_{yx}(\mu_X - \overline{X}_i). \quad (10)$$

In equation (10) b_{yx} is the slope $(b_{yx} = \frac{\rho_{yx}\sigma_Y}{\sigma_X})$ mean and variance of the S_i estimator are given as,

$$\mu_S = \mu_0, \sigma_S^2 = \frac{\sigma_Y^2}{n}(1-\rho_{YX}^2). \quad (11)$$

Based on equation (10), the AHWMA statistic can be expressed as below;

$$T_i = \lambda S_i + (1-\lambda)\overline{S}_{i-1}. \quad (12)$$

Where $\overline{S}_{i-1} = \frac{1}{n}\sum_{k=1}^{i-1} S_k$, is the average of sample means of all the $(i-1)^{th}$previous observations. S_i represents the information at i^{th} regression estimate in equation (10). The variance expression of equation (12) given as.

$$\sigma_{Ti}^2 = \begin{cases} \frac{(1-\rho_{yx}^2)}{n}\lambda^2\sigma_Y^2, & \text{if i} = 1\\ \frac{(1-\rho_{yx}^2)}{n}(\lambda^2\sigma_Y^2 + (1-\lambda)^2\frac{\sigma_Y^2}{i-1}), & \text{if i} > 1 \end{cases} \quad (13)$$

The control limits based of variance provided in equation (13) of AHWMA control chart are given as;

$$\begin{aligned} LCL_i &= \begin{cases} \mu_0 - C\sqrt{\frac{\lambda^2}{n}(1-\rho_{YX}^2)}, & \text{if i} = 1\\ \mu_0 - C\sqrt{(\frac{\lambda^2}{n} + \frac{(1-\lambda)^2}{n(i-1)})(1-\rho_{YX}^2)}, & \text{if i} > 1 \end{cases}\\ UCL_i &= \begin{cases} \mu_0 + C\sqrt{\frac{\lambda^2}{n}(1-\rho_{YX}^2)}, & \text{if i} = 1\\ \mu_0 + C\sqrt{(\frac{\lambda^2}{n} + \frac{(1-\lambda)^2}{n(i-1)})(1-\rho_{YX}^2)}, & \text{if i} > 1 \end{cases} \end{aligned} \quad (14)$$

If the AHWMA statistic goes beyond the limits provided in equations (14).

3. Design Structure of Proposed chart of use of two auxiliary variable EWMA (TAEWMA) mean chart.

Let us consider three variables taken from trivariate normal distribution which are Y, X and W. Here Y is our monitoring variable and X and W are our auxiliary variables. These variables can be expressed in matrix form as $\begin{pmatrix} Y\\ X\\ W \end{pmatrix} \sim N_3\left(\begin{pmatrix} \mu_Y\\ \mu_X\\ \mu_W \end{pmatrix}, \begin{pmatrix} \sigma_{yy} & \sigma_{yx} & \sigma_{yw}\\ \sigma_{xy} & \sigma_{xx} & \sigma_{xw}\\ \sigma_{wy} & \sigma_{wx} & \sigma_{ww} \end{pmatrix}\right)$.

The trivariate normal regression-type estimator proposed by Kadilar and Cingi (2005) is as follows,

$$R_i = \overline{y} + b_{yx}(\mu_x - \overline{x}) + b_{yw}(\mu_w - \overline{w}). \quad (15)$$

Where $\left(b_{yx} = \frac{s_{yx}}{s_{xx}}\right)$ and $\left(b_{yw} = \frac{s_{yw}}{s_{ww}}\right)$ both are regression coefficient. Further s_{yx} and s_{yw} are the sample covariance between Y, X and Y, W respectively while s_{xx} and s_{ww}both are the sample variances of X and W respectively. The $E(R) = \mu_R = \mu_0$ and $V(R) = \sigma_R^2 = (1 - \rho_{yx}^2 - \rho_{yw}^2 + 2\rho_{yx}\rho_{yw}\rho_{xw})\frac{\sigma_Y^2}{n}$, are respectively. The proposed charting statistic using equation (16) becomes as,

$$T_i = \lambda R_i + (1 - \lambda)T_{i-1}. \quad (16)$$

The control limits of the proposed TAEWMA chart can be defined as,

$$\left.\begin{aligned} LCL &= \mu_0 - K_1\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\sigma_R^2}, \\ CL &= \mu_{0,} \\ UCL &= \mu_0 + K_1\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\sigma_R^2}, \end{aligned}\right\} \quad (17)$$

In equation (17) we have described the control limits of proposed chart TAEWMA. Where K_1 is width of control limits of the proposed structure. If plotting statistic of the TAEWMA method goes beyond the control limits process is said to be OOC otherwise IC.

For calculating the RL characteristics of the proposed TAEWMA scheme codes are developed in R programming language and 50000 Monte Carlo simulations were carried out.

4. Evolution of the proposed TAEWMA chart

In this section of the article, RL performance of the proposed TAEWMA chart is reported considering the cases of multicollinearity and without multicollinearity between the two auxiliary characteristics.

4.1. Performance of the proposed TAEWMA chart without Multicollinearity

This section presents the performance of our proposed TAEWMA chart when both the auxiliary variables have their partial effect on the study variable. It's means there exists no linear relationship between the auxiliary characteristics such that ρ_{XW}=0. The ARL values of the proposed TAEWMA chart without multicollinearity case at various combinations of the design parameters are provided in Table 1. It is observed that at small choices of λ, high values of ρ_{YX} and ρ_{YW}, the proposed chart becomes much sensitive to address small shifts in the process mean. For example, at $\delta = 0.03$, λ =0.03, ρ_{YX} =0.25 and $\rho_{YW} = 0.50$, the proposed TAEWMA chart provides ARL_1=432.77 and at λ =0.03, ρ_{YX} =0.75 and $\rho_{YW} = 0.50$, the proposed structure yields $ARL_1 = 323.74$ respectively (cf. Table 1)

Table 1. **ARL of the proposed TAEWMA chart Without multicollinearity at various choices of design parameters**

δ	**Small Shifts**						**Moderate Shifts**			**Large shifts**		
	λ =0.03, K_1 = 2.483			**λ =0.05, K_1 = 2.639**			**λ =0.10, K_1 = 2.824**			**λ =0.25, K_1 = 3.001**		
	ρ_{yx}=0.25	**ρ_{yx} =0.5**	**ρ_{yx}=0.75**	**ρ_{yx}=0.25**	**ρ_{yx}=0.5**	**ρ_{yx}=0.75**	**ρ_{yx}=0.25**	**ρ_{yx}=0.5**	**ρ_{yx}=0.75**	**ρ_{yx}=0.25**	**ρ_{yx}=0.5**	**ρ_{yx}=0.75**
	ρ_{yw}= 0.50, ρ_{xw}= 0			**ρ_{yw}=0.50, ρ_{xw}= 0**			**ρ_{yw}=0.50, ρ_{xw}= 0**			**ρ_{yw}=0.50, ρ_{xw}= 0**		
0	500.72	500.78	500.73	500.78	500.13	500.40	499.82	500.52	501.54	500.59	499.91	500.83
0.03	432.77	417.12	323.74	443.59	432.81	354.67	467.98	450.11	392.93	480.23	477.92	440.92
0.05	356.55	319.50	201.35	376.43	347.81	232.23	416.38	389.84	279.31	452.20	436.79	360.55
0.075	262.07	222.49	118.31	290.58	252.76	139.54	343.11	303.55	181.56	407.90	379.11	267.50
0.1	188.41	154.90	76.21	220.32	182.74	88.71	271.41	227.55	117.64	352.12	316.26	190.05
0.125	143.22	113.66	52.92	167.39	134.62	61.19	214.71	176.80	80.58	303.48	259.53	136.73
0.175	87.28	66.49	30.16	103.41	79.02	34.19	134.76	104.65	43.34	212.87	170.36	73.29
0.2	70.33	54.16	24.30	82.65	63.41	27.24	109.55	83.37	33.64	176.88	141.29	56.29
0.25	49.14	37.75	16.66	57.07	42.91	18.56	74.19	55.19	21.98	125.53	94.39	34.81
0.5	15.48	11.84	5.26	17.29	13.13	5.78	20.20	15.27	6.43	31.05	22.05	7.77
0.75	7.82	6.08	2.78	8.68	6.63	3.00	9.84	7.37	3.33	12.80	9.17	3.73
1	4.88	3.80	1.84	5.38	4.17	1.98	5.99	4.60	2.14	7.15	5.33	2.34
1.5	2.61	2.06	1.17	2.81	2.23	1.21	3.10	2.43	1.28	3.46	2.67	1.35
2	1.75	1.43	1.02	1.86	1.52	1.03	2.02	1.63	1.04	2.21	1.74	1.05

4.2. Performance of the proposed TAEWMA chart with Multicollinearity

The term multicollinearity is described as a situation in which there exists some linear relationship between two or more variables (cf. Hawking (1983)). Usually this relationship may occur due to lack of understanding or by mistakes. Multicollinearity may happen during model specification, data collection method employed, constraints the model or in the population and over define the model. For assessment of multicollinearity on the proposed chart it is necessary to know that how every independent variable X is predicted from the other W variable. The proposed TAEWMA control chart is performed at three different stages such that ρ_{xw} ∈0.05, 0.15

and 0.25 (cf. Table 2). From Table 2 it is observed as value of ρ_{xw} increases, performance of the proposed TAEWMA chart reduces. For example, at ρ_{xw} = 0.05, λ = 0.05 at δ = 0.05, the TAEWMA chart provides ARL_1= 380.28 and at ρ_{xw} = 0.25, λ = 0.05 it gives ARL_1= 388.56 respectively (cf. Table 2).

Table 2. **RL profiles of Proposed TAEWMA chart with effect of multicollinearity at various choices of design parameters. When ρ_{yx}= 0.25, ρ_{yw} = 0.50**

δ	ρ_{xw} = 0.05			ρ_{xw} = 0.15			ρ_{xw} = 0.25		
	λ = 0.05, K_1 = 2.887			λ = 0.05, K_1 = 3.735			λ = 0.05, K_1 = 3.337		
	ARL	SDRL	MDRL	ARL	SDRL	MDRL	ARL	SDRL	MDRL
0.00	500.46	516.20	338.00	499.65	512.31	341.00	500.03	517.26	341.00
0.03	448.23	457.15	311.00	448.54	461.34	307.00	452.43	463.43	310.00
0.05	380.28	389.59	259.00	385.48	393.71	263.00	388.56	398.29	266.00
0.075	293.71	298.25	201.00	301.65	305.99	206.00	300.55	302.64	208.00
0.1	222.99	221.53	158.00	226.21	225.28	158.00	232.85	229.36	164.00
0.125	169.18	164.76	120.00	173.46	168.59	123.00	179.30	175.82	126.00
0.175	104.22	97.51	76.00	107.63	101.04	79.00	108.37	102.73	79.00
0.2	83.63	75.74	62.00	86.67	78.97	65.00	88.84	81.55	66.00
0.25	57.02	49.36	44.00	59.74	52.20	45.00	61.09	53.36	47.00
0.5	17.27	12.56	15.00	18.16	13.16	15.00	18.56	13.57	16.00
0.75	8.80	5.86	8.00	9.06	6.07	8.00	9.29	6.15	8.00
1	5.42	3.36	5.00	5.59	3.45	5.00	5.75	3.58	5.00
1.5	2.84	1.57	3.00	2.95	1.64	3.00	3.01	1.68	3.00
2	1.90	0.93	2.00	1.94	0.96	2.00	1.96	0.98	2.00

5. Comparison with existing schemes

In this section, we assess the performance of the proposed control chart namely TAEWMA taking ARL and percentage decrease in ARL (ARL_d) as performance metrics. Mostly IC ARL is denoted by ARL_0 and OOC by ARL_1. Following Abbas et al. (2020), ARL_d= $\left(\frac{ARL_0-ARL_1}{ARL_0}\right) \times 100\%$, a chart with large ARL_d at specific shift is considered more efficient than its competitor(s). in this section, comparison of the proposed TAEWMA chart is carried out with classical EWMA, AEWMA, HMWA and AHWMA control charts.

Table 3. **ARL values of classical EWMA and AEWMA charts.**

δ	Classical EWMA				AEWMA ρ_{yx}= 0.25			
	λ = 0.03	λ = 0.05	λ = 0.1	λ = 0.25	λ = 0.03	λ = 0.05	λ = 0.1	λ = 0.25
0	500.17	499.20	500.24	499.68	500.49	500.29	499.81	500.74
0.03	456.73	462.89	476.63	486.57	455.23	461.72	476.22	484.24
0.05	390.57	411.14	433.03	464.44	389.23	406.03	439.33	466.04
0.075	306.54	338.37	383.39	434.35	299.69	330.34	376.41	427.91
0.1	237.30	267.71	316.77	385.67	227.69	257.47	313.77	385.26
0.125	182.44	210.44	263.81	342.86	177.19	201.10	253.76	335.84
0.175	113.81	136.78	177.67	262.15	110.25	129.32	169.00	248.47
0.2	94.68	111.50	147.37	229.18	89.47	104.69	139.86	218.81
0.25	65.91	77.57	103.68	170.24	63.04	73.52	96.90	159.46
0.5	21.22	23.68	28.78	47.21	20.03	22.33	27.15	44.32
0.75	10.77	11.92	13.54	19.03	10.22	11.31	12.81	18.02
1	6.62	7.33	8.20	10.42	6.29	6.90	7.76	9.75
1.5	3.47	3.79	4.18	4.78	3.29	3.57	3.94	4.56
2	2.24	2.43	2.66	2.95	2.14	2.31	2.54	2.79
L	2.483	2.639	2.824	3.001	2.483	2.639	2.824	3.001

5.1 Classical EWMA and AEWMA vs Proposed TAEWMA chart;

The classical EWMA chart was suggested by Roberts (1959), to address small shifts in the process particularly. The AEWMA chart for efficient monitoring of the process mean was designed by Abbas et al. (2014). The ARL values of classical EWMA and AEWMA are presented in Table 3. At λ

= 0.05 and δ = 0.075 classical EW MA yields ARL_1= 338.37 and for AEWMA at ρ_{yx}= 0.25 it gives ARL_1= 330.34 respectively. The proposed TAEWMA chart at λ = 0.05, δ = 0.075, ρ_{yx}= 0.25 and ρ_{yw}= 0.50 produces ARL_1= 290.58 respectively (cf. Table 1). At λ = 0.03 with 3% increases in $\delta$the classical EWMA and AEWMA provide ARL_d8.65% and 8.95% and the proposed TAEWMA chart ρ_{yx}= 0.25 and ρ_{yw}= 0.50 yieldsARL_d 13.45% respectively (cf. Tables 1 and 3). Comparing Tables 1 and 3 it can be declared that the proposed TAEWMA chart has superiority over classical EWMA and AEWMA charts.

5.2 Classical HWMA and AHWMA vs Proposed TAEWMA chart

In this section the classical HWMA chart for efficient monitoring of the process mean was designed by Abbas (2018). The AHWMA chart was designed by Adegoke, Nurudeen et al. (2019) for monitoring the process mean. ARL values of the classical HMWA and AHWMA are provided in Table 4. At λ = 0.1, δ = 0.03 and classical HWMA yields isARL_1= 441.88, for AHWMA ρ_{yx}= 0.25 , λ = 0.1 andδ = 0.03 value of ARL_1= 438.71 respectively. The proposed TAEWMA chart at λ = 0.03, δ = 0.03 and ρ_{yx}= 0.25 and ρ_{yw}= 0.50 yieldsARL_1= 432.77. From Tables 1 and 4 it is obvious that the proposed designed has supremacy over HWMA and AHWMA charts.

Table 4. **ARL values of classical HMWA and AHWMA charts.**

δ	Classical HWMA				AHWMA, ρ_{yx}= 0.25			
	λ = 0.03	λ = 0.05	λ = 0.1	λ = 0.25	λ = 0.03	λ = 0.05	λ = 0.1	λ = 0.25
0	499.82	499.65	500.18	500.64	500.56	499.84	499.76	499.65
0.03	441.88	449.10	457.98	478.79	438.71	443.68	455.20	472.76
0.05	360.10	376.72	396.47	440.12	355.04	378.76	392.98	432.37
0.075	272.56	297.18	320.62	384.04	266.30	288.29	310.47	378.75
0.1	208.34	233.77	249.20	325.62	197.63	222.88	242.52	315.35
0.125	160.32	181.99	199.35	271.80	152.98	173.87	192.52	263.10
0.175	101.90	119.39	132.53	185.13	98.84	113.69	127.79	179.77
0.2	85.25	99.68	111.41	156.88	80.49	94.92	106.37	148.54
0.25	60.31	73.04	81.93	113.20	58.61	69.62	77.06	107.40
0.5	20.14	25.08	28.63	33.41	18.91	23.82	27.10	31.94
0.75	10.42	12.75	14.93	16.27	9.89	12.22	14.21	15.25
1	6.61	8.05	9.32	9.73	6.31	7.62	8.84	9.19
1.5	3.74	4.40	4.94	4.94	3.56	4.17	4.75	4.71
2	2.55	2.97	3.32	3.22	2.44	2.85	3.20	3.04
C	2.272	2.608	2.938	3.075	2.272	2.608	2.938	3.075

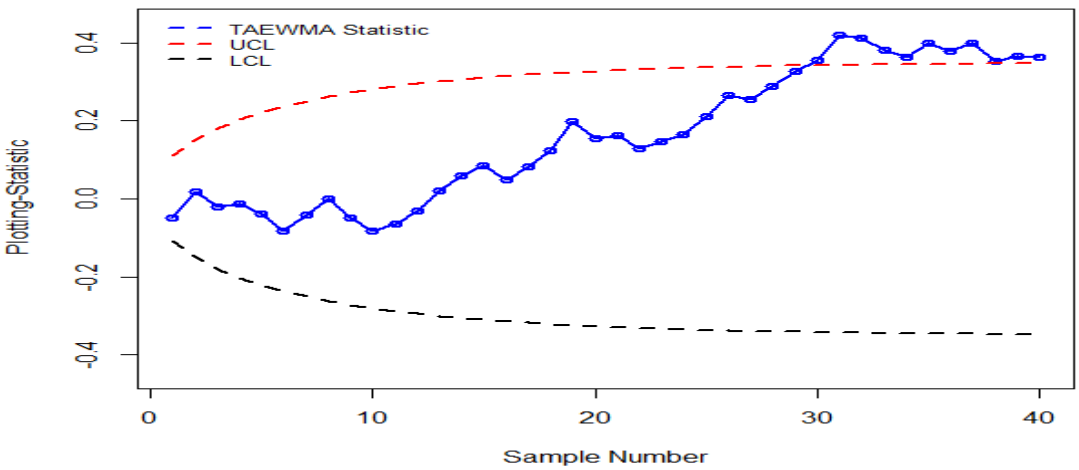

Figure 1. **proposed TAEWMA control chart without multicollinearity with design parameters ($K_1 = 2.639, \lambda = 0.05, \rho_{YX} = 0.25, \rho_{YW} = 0.50$ *and* $\rho_{XW} = 0$).**

6. Implementation of the proposed TAEWMA chart.

In this section of the article, application of the proposed TAEWMA chart is carried out using hypothetical dataset. We have considered hypothetical data set which consist of first 25 IC sample numbers and15 OOC with amount of shift 0.50 from trivariate normal distribution. The detail about the design parameters used in application for the proposed TAEWMA and its competitors are provided in Figures 1-4. From Figures 1 and 2, it can be noted that the proposed TAEWMA chart without multicollinearity traces shifts at sample number 29th, the proposed TAEWMA chart with multicollinearity traces shift at sample number 35th. AEWMA and the classical EWMA chart traces this shift at 37th and 39th sample observations respectively (cf. Figures 3 and 4). From this application it is obvious that the proposed TAEWMA chart has much better performance as compared to existing competitors particularly in case of without multicollinearity.

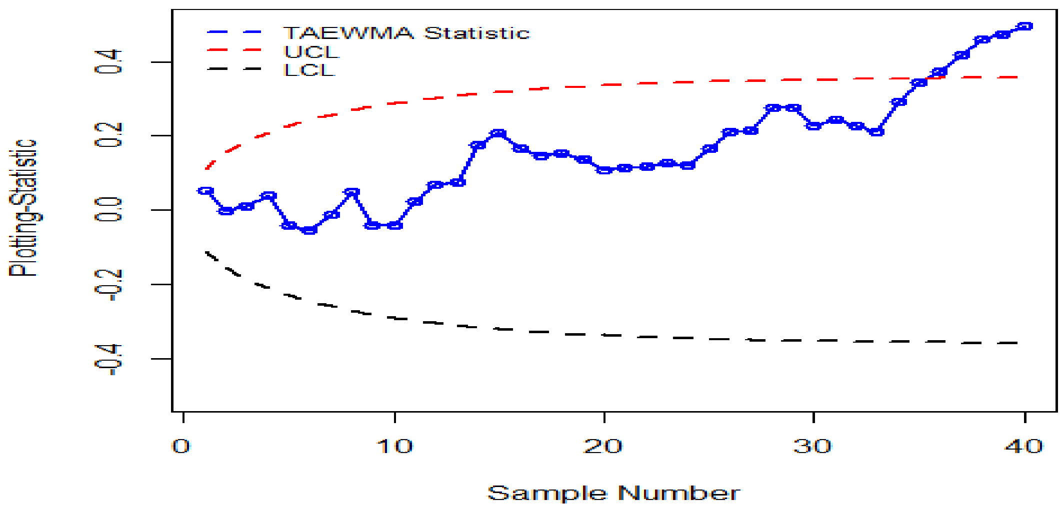

Figure 2. **Application of the proposed TAEWMA control chart with multicollinearity with design parameters ($K_1 = 2.711, \lambda = 0.05, \rho_{YX} = 0.25, \rho_{YW} = 0.50$ *and* $\rho_{XW} = 0.15$).**

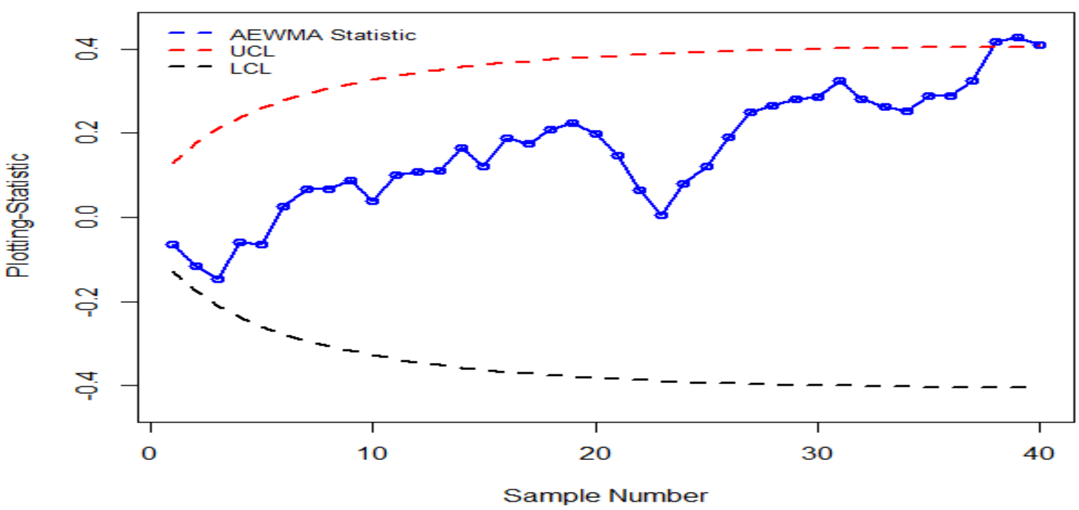

Figure 3. **Application of AEWMA control chart with design parameters ($L = 2.639, \lambda = 0.05$ *and* $\rho_{YX} = 0.25$).**

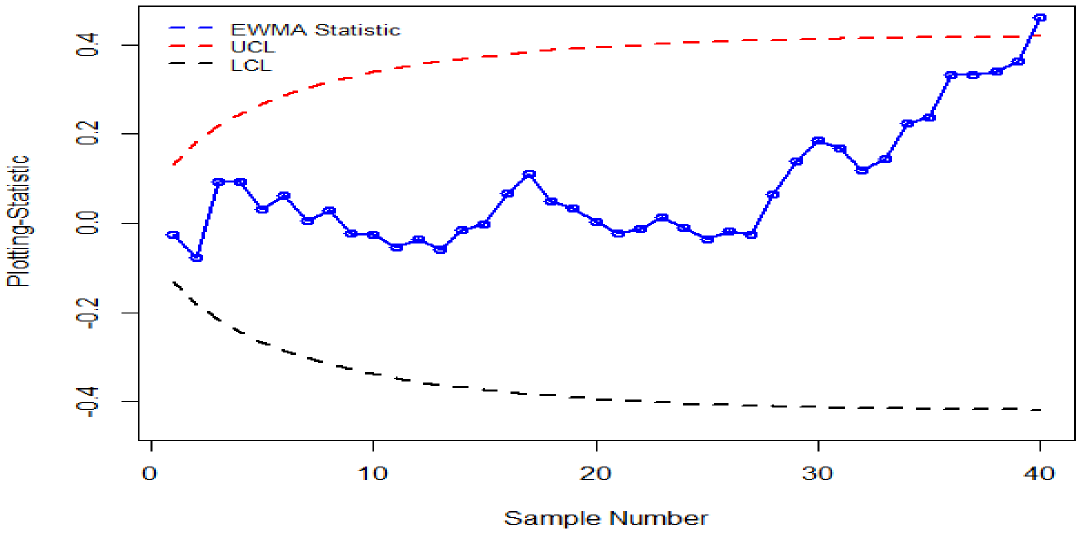

Figure 4. **Application of classical EWMA control chart with design parameters ($L = 2.639$ *and* $\lambda = 0.05$).**

7. Summary and conclusion

variation always exists in the finished products. The variation in the process output can be classified into two major categories common and special causes of variation. For improving the process output, special causes of variation should be carefully handled. For timely notifying special cause of variation, control charting schemes are used commonly. For tracing small shifts in the process mean EWMA chart is preferred due to its simplicity. In this article, we have proposed TAEWMA chart based on regression estimator using two auxiliary variable (TAEWMA) for improving the detection ability of EWMA chart. In this study, cases of multicollinearity and without multicollinearity are also incorporated. It is noted that the proposed structure has high detection ability when there exists no multicollinearity among the auxiliary variables. The proposed chart performs significantly better than the existing charts at higher choices of correlation coefficients between monitoring and auxiliary variables. An illustrative example is also included to support the results. The proposed design can be extended for monitoring dispersion and multivariate cases.

Reference

[1] Montgomery, D. C. (2012). *Introduction to statistical quality control*. (6th ed.). New York, NY: John Wiley & Sons.

[2] Page, E. S. (1954). Continuous inspection schemes. *Biometrika*, *41*(1/2), 100-115.

[3] Roberts, S.W. (1959). Control chart tests based on geometric moving average. Technometrics, 1;239-250.

[4] Abbas, N., Riaz, M., & Does, R. J. (2014). An EWMA-type control chart for monitoring the process mean using auxiliary information. *Communications in Statistics-Theory and Methods*, *43*(16), 3485-3498.

[5] Mandel, B. J. (1969). The regression control chart. *Journal of Quality Technology*, *1*(1), 1-9.

[6] Zhang, G. X. (1985). Cause-selecting control charts–a new type of quality control charts. *The QR Journal*, *12*(4), 221-225.

[7] Riaz, M. (2008). Monitoring process mean level using auxiliary information. *Statistica Neerlandica*, *62*(4), 458-481.

[8] Riaz, M. (2008). Monitoring process variability using auxiliary information. *Computational statistics*, *23*(2), 253-276.

[9] Riaz, A., Noor-ul-Amin, M., Shehzad, M. A., & Ismail, M. (2019). Auxiliary Information Based Mixed EWMA–CUSUM Mean Control Chart with Measurement Error. *Iranian Journal of Science and Technology, Transactions A: Science*, *43*(6), 2937-2949.

[10] Haq, A., & Bibi, L. (2019). The dual CUSUM charts with auxiliary information for process mean. *Communications in Statistics-Simulation and Computation*, 1-26.

[11] Abbas, Z., Nazir, H. Z., Akhtar, N., Riaz, M., & Abid, M. (2020). On designing a progressive mean chart for efficient monitoring of process location. *Quality and Reliability Engineering International*.

[12] Abbasi, S. A. (2020). Efficient Control Charts for Monitoring Process CV Using Auxiliary Information. *IEEE Access*, *8*, 46176-46192.

[13] Abbas, N., Raji, I. A., Riaz, M., & Al-Ghamdi, K. (2018). On designing mixed EWMA dual-CUSUM chart with applications in petro-chemical industry. *IEEE Access*, *6*, 78931-78946.

[14] Abbas, N. (2018). Homogeneously weighted moving average control chart with an application in substrate manufacturing process. *Computers & Industrial Engineering*, *120*, 460-470.

[15] Adegoke, N. A., Smith, A. N., Anderson, M. J., Sanusi, R. A., & Pawley, M. D. (2019). Efficient homogeneously weighted moving average chart for monitoring process mean using an auxiliary variable. *IEEE Access*, *7*, 94021-94032.

[16] Cochran, W. G. (1977). Sampling techniques (3rd ed.). New York, NY: John Wiley & Sons.

[17] Kadilar, C., & Cingi, H. (2005). A new estimator using two auxiliary variables. *Applied Mathematics and Computation*, *162*(2), 901-908.

[18] Hocking, R. R., & Pendleton, O. J. (1983). The regression dilemma. *Communications in Statistics-Theory and Methods*, *12*(5), 497-527.

[19] Chen, J. H., & Lu, S. L. (2020). A New Sum of Squares Exponentially Weighted Moving Average Control Chart Using Auxiliary Information. *Symmetry*, *12*(11), 1888.

[20] Amir, M. W., Raza, Z., Abbas, Z., Nazir, H. Z., Akhtar, N., Riaz, M., & Abid, M. (2020). On increasing the sensitivity of moving average control chart using auxiliary variable. *Quality and Reliability Engineering International*.