

Complementary and Substitute Products for Imperfect Production, Defective items with Advertisement-Sales Return-Cournot's Linear Inverse Demand Decision on: Optimal Demand and Green Production Replacement Policy

V. Choudri^{1*}, C. Senthilkumar², C.K. Sivashankari³

Abstract

The mathematical relationship that describes price as a function of quantity required is called an inverse demand function, or price function, in field of economics. In consumer decision theory, this function also reflects psychological responses to price, product perception, and purchasing intentions. A product that might be used in place of another to satisfy the same needs of the consumer is called a substitute. Complementary products, like gasoline, SIM cards, and baking powder, are items that are purchased in addition to the main product and frequently offer little value or are not used separately. This paper explores optimal demand and green production replacement models for complementary and substitute products within an imperfect production system, integrating both economic and psychological drivers of consumer demand. It examines defective items considering advertisement-influenced sales returns and Cournot's inverse linear dependent demand. It also focuses on applying a third-order equation to determine the ideal demand and green production lot size. Two models have been developed. In model 1, complementary products for imperfect production systems and defective items with advertisement-sales return are subjected to Cournot's inverse linear dependent demand; in model 2, substitute products are used to replace defective items with advertisement-sales return and imperfect production systems. The law of demand is validated, the price break-even point is determined, and the maximum profit from all three distinct product demands in both models is calculated. Both methodologies employ a carbon pricing mechanism and a green technology investment cost to reduce carbon emissions. The purpose of the study is to determine optimal manufacturing lot size and pricing to optimize total profit. Numerical examples demonstrate this concept's applicability. It is demonstrated that the degree of complementarity and substitutability affects the total profit. A sensitivity analysis of the optimal choices is given to assess the stability of proposed model. Employing Visual Basic 6.0, the data is produced.

Key words: Substitute products, Production lot size, and Law of demand, Complementary products, Defective items, Optimal demand, Price break-even point.

¹R.M.D. Engineering College (Autonomous), Chennai - 601 206, India. Email Id drchoudri@gmail.com

²Jeppiaar Institute of Technology, Sriperumbudur - 631 604, India. Email Id csenthilkumarmaths@gmail.com

³R.M.K. Engineering College, Chennai - 601 206, India. Email Id: sankariooty@gmail.com

1.INTRODUCTION

By expressing price as a function of quantity and demonstrating how much consumers are willing to pay for each unit, the inverse demand function explains how the price of an item varies with the quantity required. Typically written as $P = f(Q)$, it plays a central role in competitive market analysis by helping determine equilibrium prices and quantities across different market structures. Its applications are broad: inverse demand functions are used to derive individual firm demand curves in oligopolistic markets, informing pricing strategies; a common form is the linear inverse demand function $P = a - bQ$, here a & b are constant; in Cournot competition, firms rely on the inverse demand function to choose output levels based on rivals' production; in Bertrand competition, price-setting behavior is guided by inverse demand relationships, often yielding different equilibrium results; and in Stackelberg leadership models, the leader firm uses the inverse demand function to anticipate follower reactions to its chosen price or output decisions.

A substitute product can be utilized to satisfy customer's needs in place of another product. Customers frequently demand less of one alternative product when they have more of the other. This has been referred to as a negative connection among substitute product demand. Because they provide customers with additional options to suit their demands, substitute items are significant in the marketplace. The popularity of alternative items can be impacted by market conditions, like changes in consumer demand or new technologies. Here are a few instances of products that could be substituted: Margarine with butter, E-books and physical books, Purified water and spring water in bottles, Coffee with energy drinks, Purchasing Coke when Pepsi is sold out, as well as steel-toe and composite-toe boots. The primary product and complementary products are closely related. They frequently cannot be consumed by themselves. As a result, demand for the primary product drives demand for its complementary product. After selling the primary product, marketers either sell them together or advertise a related one. They can boost sales in this way. Mobile device accessories, boot polish, tennis rackets & balls, laptop bags, game consoles & games, printer cartridges, as well as gun ammunition are a few examples of complementary products that are marketed alongside their primary product.

Inventory management seeks to optimize stock levels to limit working capital investment and storage

costs, which have a significant influence on inventory decisions inside any firm. Inventory turnover and other performance metrics have been planned, coordinated, and regulated by inventory managers for the past century. Since Parlar's first publication in 1988, numerous studies have focused on modeling different inventory management issues involving complementarity and product replacement. When 2 free products are provided, there's a positive relationship among their requests, and to make the most of the products, buying one of the free things requires or motivates buying the other (Yue et al. 2006). A two-stage, multi-product supply chain. Pasternack & Drezner (1991) studied a stochastic single-period substitution problem, Parlar and Goyal (1984) increased it to a 2-substitutable-product scenario, and McGillivray & Silver (1978) introduced substitutes in the inventory concept. A substitution model based on shortage was presented by Drezner et al. (1995), who examined 3 different situations: full, partial, or no substitution. Inventory, production, price, and transportation policy optimization were all studied together by Dehghanbaghi and Sajadieh (2017). Taleizadeh et al. (2020) studied inventory and pricing strategies for complementary products to determine optimal order quantities and selling prices (SP) inside an inventory model employing bundle and mixed-bundle selling methods for two complementary items. They also examined 3 pricing-inventory models. Wang et al. (2017) looked at the pricing or service options of supplemental items in a dual-channel distribution system, encompassing an online channel or a traditional retail channel. To learn more about how complementary product price is determined, see Edalatpour & Al-e-Hashem (2019) along with their references. An integrated 2-stage approach with 1 vendor and one customer for 2 complimentary items has been proposed by Hemmati et al. (2017). Two complimentary deteriorating items were examined "by Sarkar et al. (2024) in a 2-echelon supply chain management. The rate of deterioration of the complementary products is taken as a function of time, and it increases over time. A degrading inventory model for complementary and substitute products was presented by Maity and Maiti (2009), who used optimal control theory to tackle the problem for both the stable and transitional scenarios. The research demonstrates that retail strategies, like pricing and ordering cycle length, must be suitably aligned with changes in the degree of complementarity.

Pricing" is one of the main strategies for raising profits in competitive marketplaces. Businesses risk losing clients to competitors if they set extravagant

prices. On the other hand, if companies set low prices, they might not make the most of the market. Businesses must balance these two tactics to avoid losses. Product promotion improves brand recognition and consumer accessibility in different markets, which impacts inventory management. Advertisements often include product information, particularly when manufacturers and suppliers release new or improved models. This information helps customers use the “product. Thus, the demand for a specific product is directly influenced by the effectiveness of marketing. To increase the number of people who buy their products, manufacturers and suppliers aim to advertise in popular media, including print and electronic media, among other channels, using modern technology and well-known personalities. Taleizadeh and Charmchi (2015) study a two-tier supply chain that includes a factory and a single retailer selling two complementary commodities, and the demand for every product is determined by both pricing and advertising. Taleizadeh et al. (2019) investigated pricing and inventory decisions for deteriorating complementary or substitutable commodities. They looked at four scenarios where the demand for one product is affected by the price of” another. Malcolm Brady (2021) investigates the behaviour of two businesses engaged in a duopoly, in which businesses can affect demand by employing advertising. Two inventory models using a demand function that incorporates a nonlinear rise impacted by advertising and a linearly decreasing price dependency were presented by Khan et al. (2023).

Production defects occur during manufacturing and fail to fulfill specifications for design. Surface flaws and limitations of material may “affect a product's safety, performance, durability, and aesthetic appeal. Incorrect assembly, inferior materials, poor machining, programming mistakes, and missing parts are examples of common defects in manufacturing. Unplanned manufacturing errors are often caused by human or machine failure. Flaws can be divided into two categories: spot faults, which are smaller and more challenging to fix, and global flaws, which are larger and more accessible. To enhance quality and lower costs, manufacturers must focus on process control and identifying the sources of variation” within “the process. De et al. (2016) examined a multi-item, EPL (imperfect economic production lot-size) model in which the items are complementary or substitutes for one another, and the SP and advertising/promotional cost depend on demand and partially rework under budgetary constraints. Chen et al. (2019) studied a joint replacement problem involving shortages and partial

demand substitution through developing a mixed integer nonlinear programming model involving damaged products. Karmakar et al. (2021) presented a detailed inventory model for substitutable products including two suppliers, each offering unique branded products with corresponding demand. All” of these products are sold through a single retailer, who also has the power to control production and defective goods.

While numerous studies explore complementary and substitutable products, none have simultaneously analyzed optimal demand and green production lot size as joint decision variables using Cournot's inverse linear dependent demand in an imperfect production system with defective items, modelled through a third-order equation. This paper discusses 2 models: model 1, which uses complementary products to replace defective items, and model 2, which uses the “law of demand” to determine “price break-even point” and calculate the maximum profit from three distinct product demands. To reduce carbon emissions, use carbon or green investment technology costs. The model's applicability is demonstrated by numerical examples. This study seeks optimal pricing or production lot size to optimize profit. The stability of the proposed model is assessed by sensitivity analysis of optimal solutions. Data is created with Visual Basic 6.0.

Background and motivational support (Contribution of this research)

This study creates a comprehensive model, driven by the significance of EPQ model and mentioned realistic scenarios. The following are this model's primary contributions:

In this study, advertisement-Cournot's Linear price-Sales return are considered as a demand function for imperfect production system for complementary and substitute products.

Two models are developed for substitute and complementary products and in the two models and additionally, it focuses on determining “the optimal demand and green production lot size using a third-order equation.

Pricing policy has always been recognized as a vital tool for attaining profit maximization in firms. Consequently, the price break-even point, the law of demand, and the highest possible profit were assessed based on demand for 3 products in both models.

To lower carbon emissions, green technology investment costs and carbon emission costs are” introduced.

To my knowledge, no author has been regarded as an advertisement-cournot's inverse price-sales return for complementary and substitute products based on their previous merits. A company that produces computers (software), printers (ink), cars (petrol), and other items could use the suggested inventory model to manage stock of a variety of goods that have production issues, namely bags, food, stationery, or clothing (substitute products).

This paper's remaining "is organized as follows: Section 2 reviews the literature; Section 3 defines the problem, its notation, and its assumptions; Section 4 developed mathematical modeling based on complementary products for defective items and imperfect production systems with advertisement-sales return-Cournot's linear inverse dependent demand; and Section 5 describes mathematical modeling for substitute products for defective items and imperfect production systems with advertisement-sales return-Cournot's linear inverse dependent demand. Section 6 indicating managerial implications, Section 7 gives comparative study between substitute and complementary products and section 8 summarizes accomplishments of proposed model.

2.REVIEW OF LITERATURE

The subsequent paragraph presents a significant overview of the significant research on production systems. An economics-based model, introduced by Coughlan (1987), shows incentives for the distribution function's integration or non-integration based on product demand complementarity, economies of scale & scope in distribution, and assessment of the customers of the advantages of 'one-stop shopping.' The seasonal fruit may be substituted with another fruit depending on availability and cost, as it is a replaceable product. When the rising price of one product correlates with a rise in the sales rate of another product, such items are considered substitutes for one another (Shocker et al. 2004). Raising the price of one product may enhance the sales of a competing one. Like, Coca-Cola, as well as Pepsi, are two substituted items, as both fulfill the same function. A customer can choose Pepsi over Coca-Cola under any circumstances. Wang (2006) studied producers who sell different products directly or through a shared reseller. The products are complementary since they are always sold and consumed together or in sets of one. Zhao et al. (2012) explored the pricing dynamics of substitutable products in a fuzzy supply chain with two competitive manufacturers producing distinct items and a common retailer selling them to consumers. Wei et al. (2013) examined price

difficulties for two complementary items in a supply chain with two producers and one retailer. In their analysis of a dynamic game of Cournot competition, Bischi et al. (2015) examine how heterogeneous companies decide on their output strategies by weighing two alternative adaptive behavioural rules. Giri et al. (2016) explored a rivalry among one alternative and one complementary product in a 2-echelon supply chain system where one producer produces one of three products and a common retailer markets those products. According to Sarkar and Lee (2017), a 2-echelon supply chain model consists of 2 producers and a common retailer. Two complimentary product kinds are produced by the producers, which the typical retailer buys individually at a reservation price and then bundles for sale. It is assumed that the demands of the manufacturers and the retailer are stochastic. In their 2017 study, Mukhopadhyay et al. examined a duopoly market in which two distinct companies provide complementary commodities in a leader-follower fashion. Each company has its own forecast data regarding the erratic market demand and chooses whether to share it with the other company. Benkherouf et al. (2017) formulated a finite horizon inventory management issue for 2 substitutable products, with demand expected to fluctuate over time and ordered concurrently throughout each replenishment period. Bian et al. (2018) use the gloomy trigger method to apply repeated game theory to tacit cooperation in dynamic distribution systems. We look at the strategic tools that rivals use in distribution networks that consist of two manufacturers each distributing via two independent shops. in a duopoly that is differentiated. Two complementary products are sold by a manufacturer to a retailer. Perlman (2019) investigated an inventory model as per base stock replenishment in the context of stochastic consumer demand, as well as the best levels of stock for two vertically differentiated commodities in both centralized and competitive environments. Taleizadeh et al. (2019) evaluated price and inventory decisions for decaying complimentary and substitutable commodities in a supply chain with two suppliers and one retailer. Four models are examined in which the demand for one product depends on the SP and the price of another product. Jalali et al. (2020) determined the best closed-loop supply chain design with complementary items. Retailer that sold 2 complementary products was taken into consideration by Poormoaid and Atan (2020). Customers arrive in accordance with a Poisson Process, unmet demands are lost, and the retailer applies a continuous review (Q,r) policy to maintain the inventory of each product. The products are demanded

either individually or collectively. In their 2020 study, Shan et al. looked at how to bundle and price complementary goods in an eco-friendly supply chain. They created three pricing models: one that focused on “separate pricing,” another on “pure bundling pricing,” and a third on “mixed bundling pricing.” By a prolonged two-stage game and a two-stage game, Leonardos (2020) investigated the impact of endogenous cost formation in the classic Cournot oligopoly and studied its equilibrium under 2 distinct information levels: complete as well as incomplete data on the supplier's side regarding the original demand of the market. In “a DCSC (dual-channel supply chain) with two manufacturers and one retailer, Ren et al. (2020) examined a price game and service collaboration for supplemental products where the products of the two producers are complementary. An analytical game-theoretical study was conducted by Gupta et al. (2021) to examine how pricing strategies for alternative items are affected by the timing of supply capacity disruptions. They” determined the best pricing and order quantities based on interruption time and product substitution. In a two-period decision framework, Saha et al. (2021) investigated the effects of strategic inventory on a supply chain that included two producers and a common retailer. In a two-period decision framework, Nielsen and Moon (2021) investigated the impact of “strategic inventory on a supply chain encompassing 2 producers and a common retailer. In each period, participants set their prices to maximize profits. Wang and Zhang (2021) analyzed a supply chain system with a manufacturer, a capital-rich retailer, or a capital-constrained retailer, including strategic relationship and substitute product competition. The production cost is minimal, and the retailers' equilibrium order numbers are based on their optimal equilibrium profits, if not on the” producer's optimal balance profits and those of the dominating retailer. To automatically identify complements as well as alternates from sales transaction data, Tian et al. (2021) looked at a network perspective. We employ a “bipartite representation of a product-purchase network, comprising transaction and product nodes, to formulate appropriate null models for inferring meaningful relationships” between products—either as complements or substitutes—and establish metrics based on random walks to assess their value. In the context of online-to-offline commercial operations, Chen & Su (2021) investigated the cooperation of an SC handling complementary products using a variety of game theory-based techniques. In 2021, Asproudis & Eilippiadis examined the idea of abatement technology. Compared to its Cournot counterpart, they

discovered that the Bertrand firm would choose a dirtier technology. Poormoaid (2022) explored a periodic review base-stock policy for two complimentary commodities with related needs and combined replenishment. Demands are modeled utilizing a Poisson process, and any missing demand is lost. When two items have demand replacement and both substitutes have the same constant demand rates and cost structures, Teunter & Kuipers (2022) look at the best inventory management. Koren et al. (2022) presented a stockout-based and dynamic stock substitution inventory management model for competitive substitutable products. Koren et al. (2022) used sales data from an online clothes retailer offering “substitutable products to compare centralized (one store selling both items) and competitive (two retailers providing one product) models. Li et al. (2023) evaluated how economies of scale, competitor conduct, and market rivalry types” affected a manufacturer's retail outsourcing decisions. Koren et al. (2023) created a “single-period inventory-management model for a market with two substitutable commodities under stockout-based substitution, which allows customers to get a substitute when their product is not available.

In an effort to discuss related topics, Mondal and Giri (2024) integrated green innovation, remanufacturing, and product complementarity in a two-stage supply chain and a green closed-loop supply chain with two producers and a typical retailer. Haque et al. (2024) developed a sustainable production inventory system for two decaying complementary” commodities to optimize delivery quantity and lot size for decentralized and central supply chain coordination. Meza et al. (2024) developed a replacement item profit optimization model that accounts for manufacturing costs and inventory management in a competitive, time-sensitive market with limited supply and changing consumer preferences. Advertisement-Cournot's Linear price-Sales return is taken into consideration in this study as a demand function for complementary and alternative items in an imperfect manufacturing system. Both methods calculate the highest profit from three product requests, discover the price break-even point, and confirm the law of demand. To reduce carbon emissions, green technology investment expenses and prices are imposed. No author has been recognized for marketing-Cournot's inverse price-sales return for complementary and alternative products based on the above merits.

Table 1 Author's Contribution Table

S. No.	"Authors	Demand pattern	Inventory	Defective items	Complementary products	Substitute products	Bertrand's Model	Cournot's Model	Price Breakeven point
1.	De et al. 2016	Pricing	yes	yes	yes	yes			
2.	Sarkar and Lee, 2017	Pricing	yes		yes				
3.	Wang et al., 2017	Pricing	yes		yes				
4.	Benkherouf et al., 2017	Time varying demand	yes			yes			
5.	Mukhopadhyay and Goswami, 2017	constant	yes	yes		yes			
6.	Hemmati et al., 2017	Stock deperendnt demand	yes		yes				
7.	Dehghanbaghi and Sajadieh, 2017	pricing			yes				
8.	Chen et al., 2019	Partial demand	yes	yes		yes			
9.	Edalatpour and Al-e-Hashem, 2019	constant	yes		yes	yes			
10.	Taleizadeh et al. 2019	Pricing	yes		yes				
11.	Taleizadeh et al. 2020	Pricing	yes		yes				
12.	Ren et al., 2020	Pricing			yes				
13.	Asproudis and Filippiadis, 2021	constant					yes	yes	
14.	Nielsen and Moon, 2021	PDD	yes			yes			
15.	Saha et al., 2021	Pricing	yes			yes			
16.	Koren et al., 2022	Stock-based	yes			yes			
17.	Koren et al., 2023	Stock-based	yes			yes			
18.	Meza et al. (2024)	PDD	yes			yes			
19.	Sarkar et al., 2024	Constant	yes			yes			
20.	Present study	Advertisement--Sales return-Cournot's PDD	yes	yes	yes	yes	yes	yes	Yes"

3. ASSUMPTIONS & NOTATIONS

The subsequent assumptions or notations are utilized in the model.

3.1 Assumptions: The study is dependent on the subsequent assumptions.

- 1) The "model addresses the production of a single product while preventing any shortages.
- 2) The starting and ending inventory levels are zero.
- 3) The linear inverse demand function is defined to be a function of Cournot's type model and it is interpreted as $P_1 = \alpha - \beta d_1 - \gamma d_2$ by Otake and Min (1996) and Singh and Vives (1984) where P_1 is price of the seller-1, α is intercept of inverse demand function, β represent own price effect, γ is cross price effect, and d_1 and d_2 represents "per-unit-time demand of products. In this model,

the demand is defined as advertisement-Cournot's inverse demand function- sales return for complementary products such as $D(A, SP_{cp}) = A^d(a - bD_1 + cD_2)(1 + s)$ and for substitute product(sp) is $D(A, SP_{sp}) = A^d(a - bD_1 - cD_2)(1 + s)$ where A is number of advertisement and s is the rate of sales return.

- 4) According to Varian (1992), the cross-price effects must be symmetric with $b > c$ in order for a consumer demand function to function properly. Choi (1991) observed that variations among b & c is directly proportional to degree of product substitutability among 2 items.
- 5) Observed discrepancy among the unit production cost as well as the SP i.e., $Y > C_p$
- 6) Production rate is predictable along with

constant across planning horizon's codes. Costs of setting up a set, producing a unit, storing a unit for a defined amount of time, improving quality, marketing, carbon emissions, and green investment technologies are all recognized and

fixed.

3.2 Notations: The subsequent notations are used in for analysis.

"Particulars"	Notations/units	Particulars	Notations/units
Rate of Production	Y-units	Per unit advertisement cost	C_A - Money
Intercept of the demand	a - units	Quality cost/unit	C_Q - Money
Own price effect / cross demand effect	b / c - units	Per unit production cost	C_P - Money
Per set setup cost	C_o -Money	Rate of sales return"	s
Per unit/time holding cost	C_h - Money	Advertisement frequency	d
Rate of defective items	v	Number of advertisement/units	A_n - numbers
Cycle time	T	Green Technology Investment cost per unit	g_s
Carbon emission cost per unit	C_{es} Money	Maximum inventories	Q_1, Q_2
Demand for the product for seller -1/seller-2	D_1/D_2 Money	Production, Reworking, and Consumption time	T_1, T_2, T_3
Optimum quantity	Q - units	Parameter for Green Technology Investment	σ
Number of defective items: Yv units (Here Y represents production rate and v is rate of defective items. Therefore, Yv represents number of defective items.)			
Total profit function with two decision variables D_1 & Q			$TP(D_1/D_2, Q)$

4. MATHEMATICAL MODEL - COMPLEMENTARY PRODUCTS "FOR IMPERFECT PRODUCTION SYSTEM, DEFECTIVE ITEMS, WITH ADVERTISEMENT-SALES RETURN-COURNOT'S LINEAR INVERSE DEPENDENT DEMAND

Production" scheduling and planning are fundamentally dependent on the production inventory model. Producers and inventory managers use the EPQ model to determine production and cost. Without adjusting for shortages, the model considers manufacturing a single product. Routine manufacturing eliminates all defective items, and it is assumed that completed products can only be delivered to customers if the entire lot is guaranteed to be of excellent quality at the end of production. The product becomes technologically infallible toward the conclusion of production, or consumer preferences shift. The quantity of product stocks should be carefully managed. "This is how the suggested inventory system works. Cycle initiates at $T=0$ and inventory accumulate at rate of $Y(1-v) - A^d(a - bD_1 + cD_2)(1+s)$ upto

time $t=T_1$ where the production stops. After that, inventory level starts to decrease because of demand and defective at a rate of $D = A^d(a - bD_1 + cD_2)(1+s)$ upto time $t = T_2$. This" model is based on Cournot's linear inverse dependent demand for complementary products, where the demand faced by Seller 1, denoted as D_1 , is treated as that of single item in absence of allowing any shortages. Process is repeated. Inventory level's behaviour is "depicted in the following figure –1. Rate of production of good products is always $\geq$ sum of demand rate as well as rate at which defective items are generated. So, we must have $P_1 \geq D + v$ (v =defective items). Q_1 represents the quantity of good items remaining after consumption at the end of time t_1 . From Figure1, it can be shown" that

$$\text{From the } \triangle ABK, T_1 = \frac{Q}{Y} \quad (1)$$

$$Q_1 = (Y(1-v) - A^d(a - bD_1 + cD_2)(1+s))T_1$$

From the equation (1),

$$Q_1 = \frac{(Y(1-v) - A^d(a - bD_1 + cD_2)(1+s))Q}{Y} \quad (2)$$

therefore, $T_1 = \frac{Q_1}{Y(1-v) - A^d(a-bD_1+cD_2)(1+s)}$. (3)

During the consumption time T_2 , the product becomes technologically obsolete or customer tastes

change. Care should be taken to control the amount of stocks of the product. The inventory level starts to decrease because of demand at a rate of $D = A^d(a-bD_1+cD_2)(1+s)$.

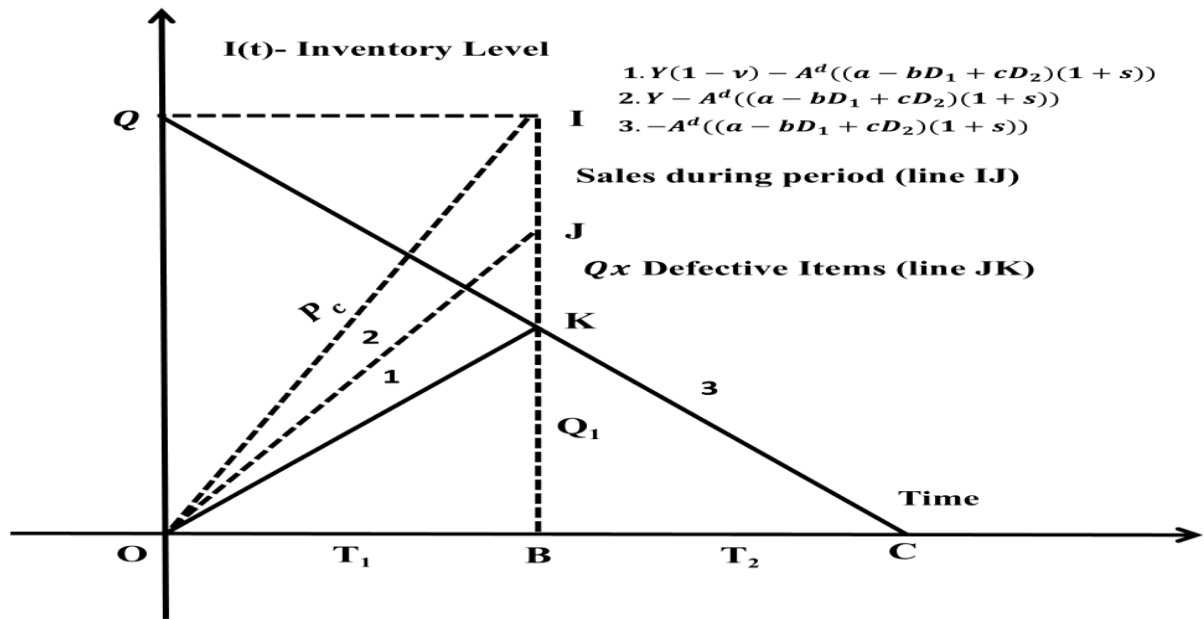


Figure 1: Production Inventory Model

From the $\triangle BKC$, $Q_1 = A^d(a-bD_1+cD_2)(1+s)T_2$,

$$T_2 = \frac{Q_1}{A^d(a-bD_1+cD_2)(1+s)} \quad (4)$$

The cycle time $T = T_1 + T_2$

$$= \frac{Q_1}{Y(1-v) - A^d(a-bD_1+cD_2)(1+s)} + \frac{Q_1}{A^d(a-bD_1+cD_2)(1+s)}$$

On simplification,

$$T = \frac{Q(1-v)}{A^d(a-bD_1+cD_2)(1+s)} \quad (5)$$

Average Inventory:

The average inventory is computed as follows from the Eqn:

$$\begin{aligned} &= \frac{1}{2T} [Q_1 T_1 + Q_1 T_2] = \frac{Q_1}{2T} \left[\frac{Q_1}{Y(1-v) - A^d(a-bD_1+cD_2)(1+s)} + \frac{Q_1}{A^d(a-bD_1+cD_2)(1+s)} \right] \\ &= \frac{Q_1^2}{2T} \left[\frac{Y(1-v)}{(Y(1-v) - A^d(a-bD_1+cD_2)(1+s))A^d(a-bD_1+cD_2)(1+s)} \right] = \\ &= \frac{(Y(1-v) - A^d(a-bD_1+cD_2)(1+s))Q}{2Y} \quad (6) \end{aligned}$$

Total cost: It consist of production "cost, holding cost, setup cost, quality improvement cost, advertisement cost, carbon emission cost, green technology cost" and all cost are known and constant.

$$\text{Production Cost} = \frac{1}{T} Q C_p = \frac{A^d(a-bD_1+cD_2)(1+s)C_p}{1-v} \quad (7)$$

$$\text{Setup Cost} = \frac{C_0}{T} = \frac{A^d(a-bD_1+cD_2)(1+s)C_0}{Q(1-v)} \quad (8)$$

$$\text{Holding Cost} = \frac{(Y(1-v) - A^d(a-bD_1+cD_2)(1+s))C_h Q}{2Y} \quad (9)$$

$$\text{Quality Improvement cost} = \frac{A^d(a-bD_1+cD_2)(1+s)vC_Q}{(1-v)} \quad (10)$$

$$\text{Advertisement cost} = \frac{A_n C_A}{T} \frac{A^d(a-bD_1+cD_2)(1+s)A_n C_A}{Q(1-v)} \quad (11)$$

$$\text{Carbon Emissions cost} = \frac{C_{es}}{T} \int_0^T (\text{Average Inventory}) dt \quad (12)$$

$$\text{Green Investment Technology cost} = \frac{\sigma g_s}{T} = \frac{A^d(a-bD_1+cD_2)(1+s)\sigma g_s}{Q(1-v)} \quad (13)$$

$$\begin{aligned} &\text{Total Profit } TP(Q, D_1/D_2) = \\ &\left[A^d(a-bD_1+cD_2)(1+s)D_1 - \frac{A^d(a-bD_1+cD_2)(1+s)(C_p+vC_Q)}{1-v} \right. \\ &\quad \left. - \frac{(C_h+C_{es})Q(Y(1-v) - A^d(a-bD_1+cD_2)(1+s))}{2Y} \right. \\ &\quad \left. - \frac{A^d(a-bD_1+cD_2)(1+s)(C_0+A_n C_A+\sigma g_s)}{Q(1-v)} \right] \quad (14) \end{aligned}$$

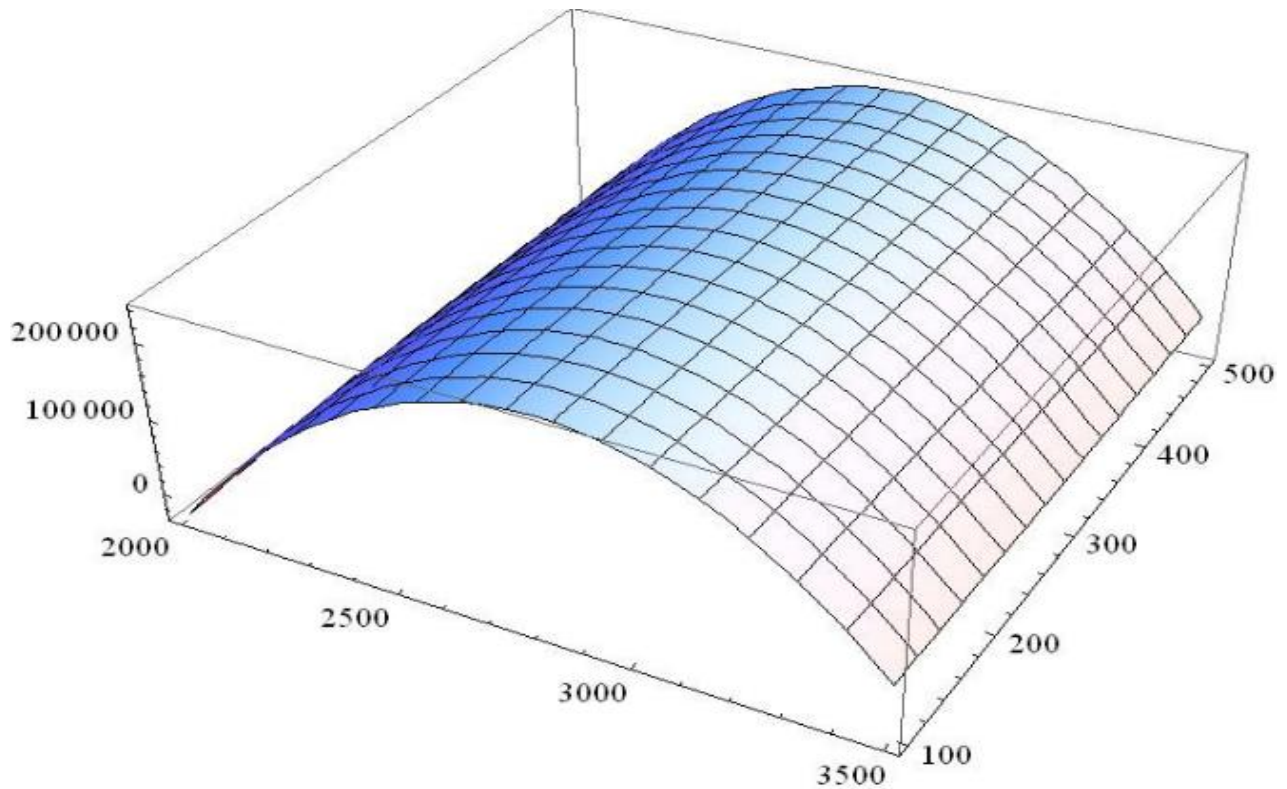


Figure 2: Profit TP (Q, D1) versus Q and D1

The above figure shows concave function of Q and D1

The above figure is of eq. (14)

{The optimal solution is TP = 241002.4261685542, {D1 → 2797.951137021527, → 216.16366225957353}}

Optimality: It can be easily shown that $TP(Q, d_1)$ is a convex function in Q & d_1 .

$$\frac{\partial}{\partial Q} TP(Q, D_1) = 0 \text{ \& \; } \frac{\partial^2}{\partial Q^2} TP(Q, D_1) < 0 \text{ and } \frac{\partial}{\partial D_1} TP(Q, D_1) = 0 \text{ \& \; } \frac{\partial^2}{\partial D_1^2} TP(Q, D_1) < 0$$

Partially differentiate the eqn (14) with respect to Q,

$$\frac{\partial}{\partial Q} TP(Q, D_1) = \left[\frac{(C_h + C_{es})(Y(1-v) - A^d(a - bD_1 + cD_2)(1+s))}{2P_c} + \frac{A^d(a - bD_1 + cD_2)(1+s)(C_0 + A_n C_A + \sigma g_s)}{Q^2(1-v)} \right] = 0$$

$$\text{and } \frac{\partial^2}{\partial Q^2} TP(Q, D_1) = \left[-\frac{A^d(a - bD_1 + cD_2)(1+s)(C_0 + A_n C_A + \sigma g_s)}{Q^3(1-v)} \right] < 0$$

On simplification,

$$Q^2 = \frac{2YA^d(a - bD_1 + cD_2)(1+s)(C_0 + A_n C_A + \sigma g_s)}{(C_h + C_{es})(Y(1-v) - A^d(a - bD_1 + cD_2)(1+s))(1-v)} \quad (15)$$

Total profit = Total sales - Total cost (sum of Eqn (6) to (12))

$$\begin{aligned} \text{Total sales} &= \text{Demand of the product} \times \text{Price per unit} \\ &= A^d(a - bD_1 + cD_2)(1+s)D_1 \end{aligned}$$

Partially differentiate the equation (14) in context of d_1 ,

$$\begin{aligned} \frac{\partial TP(Q, D_1)}{\partial D_1} &= \left[A^d(a - 2bD_1 + cD_2)(1+s) + \frac{A^d b d_1 (1+s)(C_p + v C_Q)}{1-v} \right] \\ &+ \frac{A^d b (C_0 + A_n C_A + \sigma g_s)}{Q(1-v)} \\ &= 0 \\ (a - 2bD_1 + cD_2)(1-v) + b(C_p + v C_Q) &+ \frac{b(C_0 + A_n C_A + \sigma g_s)}{Q} = 0 \\ Q &= \frac{-b(C_0 + A_n C_A + \sigma g_s)}{[(a + cD_2)(1-v) + b(C_p + v C_Q)] - 2b(1-v)D_1} \end{aligned}$$

$$\text{Therefore, } Q = \frac{-b(C_0 + A_n C_A + \sigma g_s)}{K - 2b(1-v)D_1} \quad (16)$$

here, $K = (a + cD_2)(1 - v) + b(C_p + vC_Q)$

Substitute value of eqn (16) in (15)

$$\frac{b^2(C_0 + A_n C_A + \sigma g_s)}{[K - 2b(1 - v)D_1]^2} = \frac{2YA^d(a - bD_1 + cD_2)(1 + s)}{(C_h + C_{es})(Y(1 - v) - A^d(a - bD_1 + cD_2)(1 + s))} \quad (17)$$

On cross multiplication of the above eqn,

$$2YA^d((a + cD_2) - bD_1)(1 + s)[K - 2b(1 - v)D_1]^2 = b^2(C_0 + A_n C_A + \sigma g_s)(C_h + C_{es})(Y(1 - v) - A^d((a + cD_2) - bD_1)(1 + s))(1 - v)$$

$$\text{RHS} = 2YA^d((a + cD_2) - bD_1)(1 + s)[K - 2b(1 - v)D_1]^2$$

$$= 2YA^d((a + cD_2) - bD_1)(1 + s)[K^2 - 4Kb(1 - v)D_1 + 4b^2(1 - v)^2D_1^2]$$

On simplification

$$\begin{aligned} &= 2P_c A^d(1 + s) \left[-4b^3(1 - v)^2D_1^3 + 4b^2(1 - v) \left[\frac{(1 - v)}{(a + cD_2) + K} \right] D_1^2 \right. \\ &\quad \left. - bK[4(1 - v)(a + cD_2) + K]D_1 + K^2(a + cD_2) \right] \\ &= 2YA^d(1 + s) \left[-4b^3(1 - v)^2D_1^3 + 4b^2(1 - v)[(1 - v)(a + cD_2) + K]D_1^2 \right. \\ &\quad \left. - bK[4(1 - v)(a + cD_2) + K]D_1 + K^2(a + cD_2) \right] \\ &= 2YA^d(1 + s) \left[-4b^3(1 - v)^2D_1^3 + 4b^2(1 - v)^2(a + cD_2)D_1^2 \right. \\ &\quad \left. + 4b^2C_1(1 - v)D_1^2 - 4bK(1 - v)(a + cD_2)D_1 \right. \\ &\quad \left. - bK_1^2D_1 + K_1^2(a + cD_2) \right] \\ &= \left[\begin{aligned} &-8Yb^3A^d(1 + s)(1 - v)^2D_1^3 \\ &+ 8Yb^2A^d(1 + s)(1 - v)[(1 - v)(a + cD_2) + K]D_1^2 \\ &- 2YbA^d(1 + s)K[4(1 - v)(a + cD_2) + K]D_1 + \\ &2YA^d(1 + s)K^2(a + cD_2) \end{aligned} \right] \\ &\text{RHS} = b^2(C_0 + A_n C_A + \sigma g_s)(C_h + C_{es})(Y(1 - v) - A^d((a - cD_2) - bD_1)(1 + s))(1 - v) \\ &= \left[\begin{aligned} &[Y(1 - v) - A^d(1 + s)(a + cD_2)]b^2(C_0 + A_n C_A + \sigma g_s) \\ &\quad (C_h + C_{es}) \\ &\quad (1 - v) \\ &+ b^3(C_0 + A_n C_A + \sigma g_s)(C_h + C_{es})(1 - v)A^d(1 + s)D_1 \end{aligned} \right] \\ &\text{RHS} + \text{LHS} \end{aligned}$$

$$\left[\begin{aligned} &8Yb^3A^d(1 + s)(1 - v)^2D_1^3 \\ &- 8Yb^2A^d(1 + s)(1 - v) \left[\frac{(1 - v)(a + cD_2) + K}{K} \right] D_1^2 \\ &+ \left[\begin{aligned} &2YbA^d(1 + s)K[4(1 - v)(a + cD_2) + K] \\ &+ b^3(C_0 + A_n C_A + \sigma g_s)(C_h + C_{es})A^d(1 - v) \end{aligned} \right] D_1 \\ &- \left[\begin{aligned} &2YA^d(1 + s)K^2(a + cD_2) \\ &- (Y(1 - v) - A^d(1 + s)(a + cD_2))b^2 \end{aligned} \right] \end{aligned} \right] \quad (17)$$

Here optimum solution for demand d_1 in 3rd order equation.

4.1 Numerical example-1

Optimum "solution is confirmed via numerical example with following data

Production rate (Y) = 1100 units, intercept of demand function (a) = 900 units, own demand effect (b) = 0.31, cross demand effect ϕ = 0.1, advertisement frequency (d) = 0.1, Rate of sales return (s) = 0.1, demand of the seller -2 = 2900, setup cost per set (C_0) = 700 (money value), holding cost per unit /time (C_h) = 31 (money value), carbon emission cost per unit 32 (money value), number of advertisement = 12 number, advertisement cost per unit = 40 (money value), Production cost/unit (C_p) = 2000 (money value), Rate of defective (v) = 0.01, Cost of quality improvement (CQ) = 300 (money value), Parameter for green technology investment (σ) = 8, Green technology" investment cost per unit = 80 money value.

Optimal Solution: Optimal solution for given data are

3rd order Eqn for the model for seller 1

$$362.367260401D_1^3 - 3515198.5726D_1^2 + 1126704814002D_1 - 11949713601748.9 = 0$$

The above equation contains 3 values for D_1 , that is 2926.1995, 2935.7823, 3838.6698 in which the demand 2935.78 which is reaching highest possible profit of 352635.02. The optimal demand of the product (D_1) = 2935.78 money value, Optimal quantity (Q) = 191.06 units, Optimal cycle time (T) = 0.4791 (Time), Production time (T_1) = 0.1736 (Time), Maximum inventory (Q_1) = 120.98 units, Setup cost = 1460.91 (money value), "production cost = 797480.80 (money value), Holding cost = 1869.04 (money value), Quality improvement cost = 1196.22 (money value), advertisement cost = 1001.76 (money value), carbon emission cost = 1929.32, Green Investment Technology cost = 1335.68, Total cost = 806273.75 (money value), Total" Sales = 1158908.77 (money value), Total profit = 352635.02 (money value)

Numerical example-2 (for sensitivity analysis)

Optimum solution is confirmed by numerical example with subsequent data

Production rate (Y) = 1000 units, intercept of demand function (a)= 800 units, own demand effect (b) = 0.3, cross demand effect $\odot$ = 0.1, advertisement frequency (d) = 0.1, Rate of sales return (s) = 0.1, demand of the seller -2 = 2700, setup cost/ set (C_o) = 800 (money value), holding cost per unit /time (C_h) = 20 (money value), carbon emission cost/unit 22 (money value), number of advertisement = 10 number, advertisement cost/unit = 54 (money value), Production cost/ unit (C_p) = 2000 (money value), Rate of defective (v) = 0.01, Cost of quality improvement (CQ) = 400 (money value), Parameter for green technology investment (σ) = 10, Green technology investment cost/unit = 70 money value.

Optimal Solution: Optimal solution for proposed data are

3rd order equation is the demand for

the product (D_1) for seller 1

$$293.168176353D_1^3 - 2684709.7846D_1^2 + 8137025221.68D_1 - 1173233874.27 = 0$$

Equation above comprises 3 value for D_1 , which is 2790.72, 2800.23, 3566.62 in which the demand 2800.23 which is reaching highest possible profit of

241000.28.

The optimal demand of the product (D_1) = 2800.23 money value, Optimal quantity (Q) = 215.69 units, Optimal cycle time (T) = 0.6706 (Time), Production time (T_1) = 0.2156 (Time), Maximum inventory (Q_1) = 144.85 units, Setup cost = 1192.92 (money value), production cost = 643256.63 (money value), Holding cost = 1448.55 (money value), Quality improvement cost = 1286.51 (money value), advertisement cost = 805.22 (money value), carbon emission cost = 1593.40, Green Investment Technology cost = 1043.01, Total cost = 650627.06 (money value), Total Sales = 891627.34 (money value), Total profit = 241000.28 (money value)

4.2 Determining the Price-Break Even Point

Company's safety margin should be computed and compared to its earnings and expenses for ascertaining its "Price-Break-even point". This evaluation demonstrates how much revenue is needed to meet operational costs. Break-even pricing determines a product's profit-free price. It represents income-expenditure balance. Many companies utilize break-even pricing to set product prices. The company may set a price below break even. The company would generate income but not profit in this scenario. The majority of e-commerce operations continue to lose money. E-commerce businesses would put market share ahead of profit in this scenario. For financial gain, the company could raise the product's price.

Table 5 Price Breakeven Point

D_1	Q	T	T_1 / Q_1	SC/PC	HC/ Adv.	Car/ QIC	Green TI cost	Total cost	Total sales	Total profit
2034.0423	420.53	0.6538	0.4205 148.56	1223.48 1286306	1485.68 825.85	1634.23 2572.61	1070.55	1295118	1295119	0
2000	433.95	0.6601	0.4339 147.17	1211.98 1314877	1471.70 818.09	1618.87 2629.75	1060.49	1323688	1301729	-21960
2100	396.30	0.6439	0.3963 150.86	1242.42 1230949	1508.65 838.63	1659.52 2461.69	1087.12	1239747	1279572	39824
2200	363.24	0.6333	0.3632 153.57	1263.06 1147021	1523.72 852.57	1687.09 2294.04	1105.18	1155757	1249106	93349
2300	333.67	0.6277	0.3336 154.74	1274.40 1063092	1547.48 860.22	1702.23 2126.18	1115.10	1071718	1210331	138613
2400	306.78	0.6266	0.3068 155.02	1276.66 979164	1550.23 861.75	1705.26 1958.32	1117.08	987634	1163247	175613
2500	281.98	0.6299	0.2819 154.20	1269.91 895236	1542.04 857.19	1696.24 1790.47	1111.17	903503	1107854	204351
2600	258.78	0.6379	0.2587 152.27	1254.01 811307	1522.72 846.45	1674.99 1622.61	1097.25	819325	1044153	224827
2700	236.82	0.6511	0.2368 149.18	1228.57 727379	1491.84 829.29	1641.02 1454.76	1075.00	735100	972142	237043
2800	215.73	0.6705	0.2157 144.86	1193.01 643451	1448.67 805.29	1593.53 1286.90	1040.89	650822	891823	241000

2900	195.22	0.6978	0.1952 139.20	1146.39 559522	1392.05 773.81	1531.25 1119.04	100309	566488	803194	236706
------	--------	--------	------------------	-------------------	-------------------	--------------------	--------	--------	--------	--------

d_1 -Demand, SC-Setup cost, PC-Production cost, HC-Holding cost, RWC-Reworking cost, Rej. -Rejecting cost, QIC-Quality Improvement cost, Adv. - Advertisement cost, Car-Carbon emission cost, GTI - Green Investment Technology cost, TC-Total cost, TS-Total sales, TP-Total profit

From Table 2, it is evident that cycle time as well as total profit increase as product demand rises, indicating a positive relationship. Conversely, optimum quantity, production cost, quality improvement cost, production time, maximum inventory, "setup cost, holding cost, advertisement cost, carbon emission cost, green investment cost, total cost, and total sales decrease with an increase in demand, signifying a negative relationship. Based on the given data for the product, the breakeven price/unit is 2045.1504, meaning no profit would be generated at that price. However, in a competitive market, maximizing profit requires selling fewer units at higher" price. Figure 6 shows the connection among price/unit and total profit: Break-even point in price. The graphical link

among price/unit and optimal quantity is displayed in Figure 7. It may be demonstrated that in order to maximize profit in a competitive market, a higher price must be offered in a lesser quantity. It has been observed that when price increases, quantity in units decreases. This is "law of demand," as has been established.

4.3 Law of Demand

According to rule of demand, while all other conditions stay same, quantity of a good "or service that is desired and its price are inversely related. People are less inclined to buy something when prices rise. Relationship among optimal amount and the price/unit is graphically depicted in Figure 7. It is obvious that selling more units at a higher price and fewer units at a lower price is the best method to maximize profit in a competitive market. It has been noted that when prices increase, unit quantity drops. Table 6 makes it" clear that the "law of demand" is satisfied.

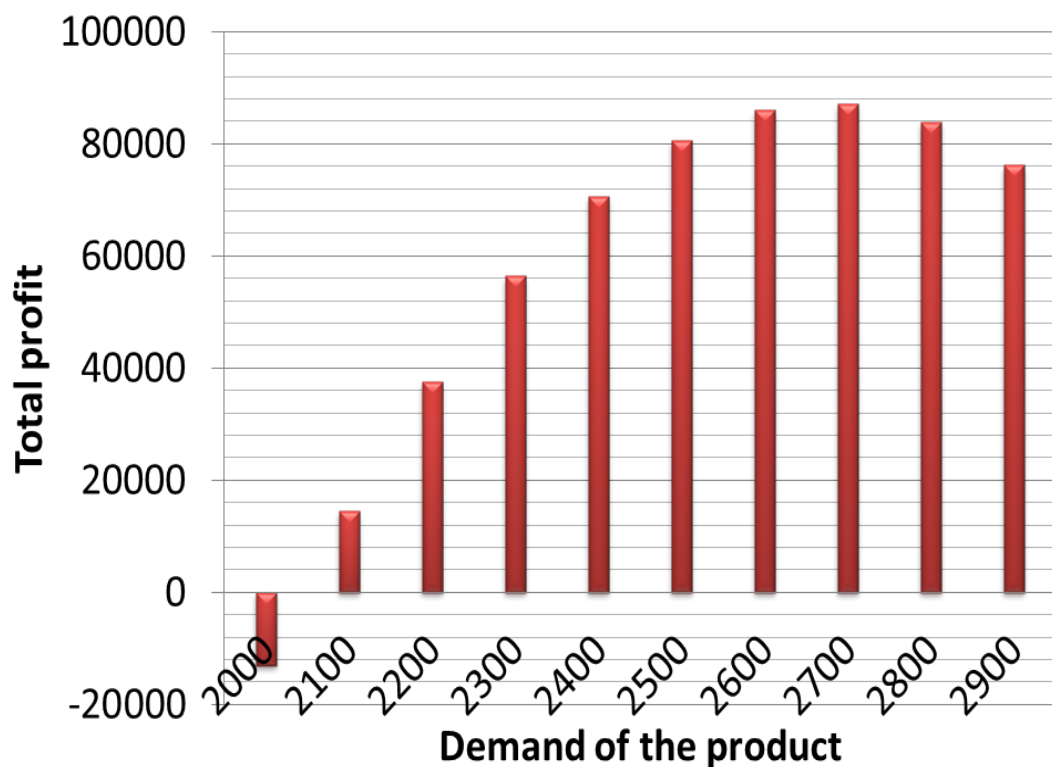


Figure 6: Price Breakeven point

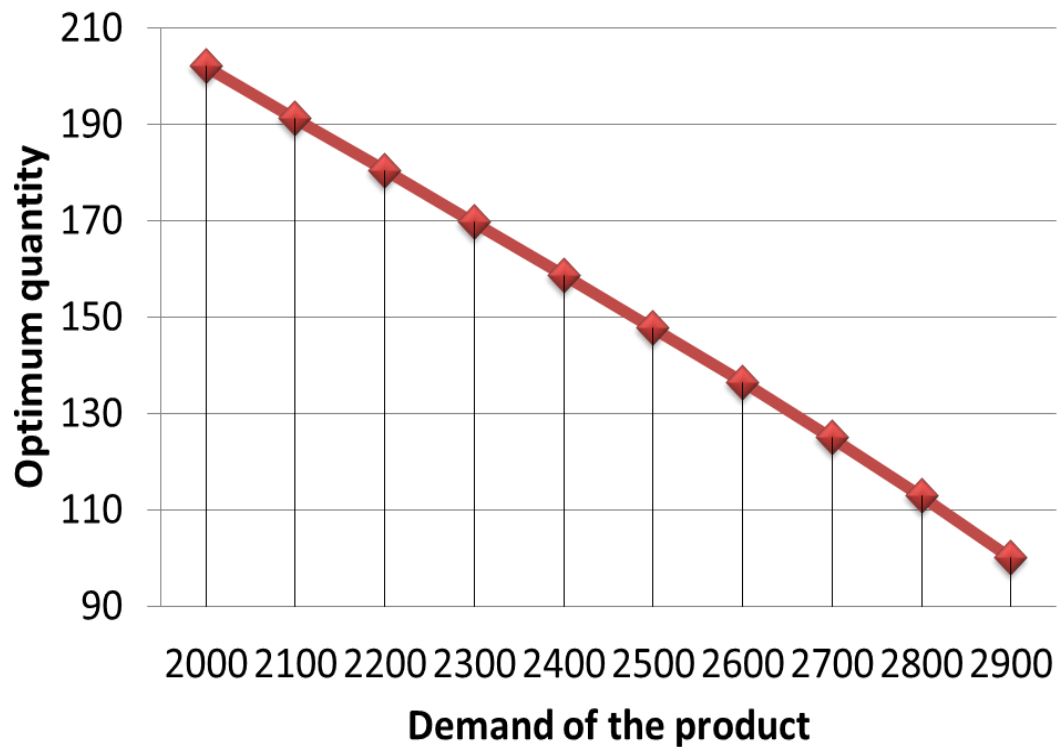


Figure 7: Relation between demand of the product and optimum quantity

4.4 Highest Possible profit

The ability of a company or firm to make the most profit while spending the least amount of money is primary objective of any organization as well as one of objectives of financial management. The term highest possible profit in financial management is a tactic or approach that increases a companies' profit or profits/share. The goal of each financial decision or investment should be to maximize earnings as much as feasible. Highest possible profit refers to method used by companies to identify the best O/P as well as pricing

points to maximize their return. To meet its profit targets, corporation makes significant adjustments to the product's cost, output volume, and sale price. Even if it hurts the customer, profit is necessary for any firm to survive.

From the third-order equation, there are three demand values: 2660.41, 2676.44 and 3312.38. The highest possible profit, 87364.67, corresponds to a demand of 2676.44. Therefore, we consider 2676.44 units as the demand for this model.

Table 6 Highest Possible Profit for three demands from the third order equation

S.No.	Items	Demand for the product		
		2660.41	2676.44	3312.38
1.	Optimum Quantity (Q)	129.49	127.62	1.62
2.	Optimum cycle time (T)	0.8872	0.8964	60.44
3.	Production time (T_1)	0.1294	0.1276	1.64
4.	Maximum inventory (Q_1)	109.48	108.35	1.60
5.	Setup cost	901.64	892.36	13.23
6.	Production cost	291887	284709.89	53.71
7.	Holding cost	1094.86	1083.58	16.07

8.	Advertisement cost	608.61	602.34	8.93
9.	Carbon emission cost	1204.34	1191.94	17.67
10.	Green Technology investment cost	789.94	780.81	11.58
11.	Quality improvement cost	583.77	569.41	0.11
12.	Total cost	297069.36	289830.36	121.32
13.	Total sales	384387.07	377195.04	88.07
14.	Total profit	87313.71	87364.67	-33.25

4.5 Sensitivity Analysis

Table 7 Effect of Demand and Inventory parameters on Optimum values in imperfect production system with rework

CP	Demand (d_1)	Q	Setup cost	Holding cost	Advertising Cost	Carbon cost	QIC	Green cost	Total cost	Total profit
intercept of the demand function (a)										
700	2367.71	86.97	658.32	799.40	444.37	879.33	286.28	576.03	146784	20979
750	2521.61	108.46	789.65	958.86	533.01	1054.75	428.26	690.94	218587	48692
800	2676.44	127.62	892.36	1083.58	602.34	1191.94	569.41	780.81	289830	87365
850	2831.70	145.54	975.92	1185.05	658.74	1303.55	710.19	853.93	360782	136951
900	2987.19	162.81	1045.07	1269.01	705.42	1395.91	850.74	914.43	431552	197429
Own demand effect (b)										
0.12	3427.51	163.43	1047.38	1271.82	706.98	1399.04	855.92	916.45	434157	291928
0.13	3227.03	156.47	1020.95	1239.73	689.14	1363.70	798.74	893.33	405380	232574
0.14	3057.47	149.43	992.45	1205.12	669.91	1325.63	741.52	868.40	376565	184565
0.15	2912.21	142.29	961.69	1167.77	649.14	1284.54	684.23	841.48	347707	145473
0.16	2676.44	127.62	892.36	1083.58	602.34	1191.94	569.41	780.81	289830	87365
Cross demand effect ©										
0.10	2676.44	127.62	892.36	1083.58	602.34	1191.94	569.41	780.81	289830	87365
0.11	2592.78	117.48	839.71	1019.65	566.80	1121.61	493.26	734.78	251406	65124
0.12	2509.20	106.85	780.38	947.61	526.75	1042.36	416.94	682.83	212869	46071
0.13	2426.03	95.53	712.59	865.29	481.00	951.82	340.38	623.52	174167	302115
0.14	2343.26	83.17	633.43	769.16	427.56	846.08	263.41	554.25	135201	17568
Advertisement frequency (d)										
0.10	2676.44	127.62	892.36	1083.58	602.34	1191.94	569.41	780.81	289830	87365

5.INTERPRETATION OF THE SENSITIVITY ANALYSIS

1. The demand for seller-1, optimal setup, quantity, holding, carbon emission, advertising, quality improvement, green investment technology, total cost, and total profit all increase in tandem with demand intercept.
2. The seller-1's demand, ideal quantity, setup and holding expenses, carbon emission costs, advertising costs, quality improvement costs, green investment technology costs, total costs, total profits all decrease as the own price impact increases.
3. Demand for seller-1, optimal quantity, setup, holding, carbon emission, advertising, quality enhancement, green investment technology, total cost, and total profit all decrease as the cross price impact rises.
4. As number of ads rises, the demand for seller-1, setup cost, green investment technology cost all decrease, while the ideal quantity, holding cost, setup cost, carbon emission cost, quality improvement cost, advertisement cost, green investment technology cost, total cost, and total

profit all increase.

5. As rate of sales return rises and demand for seller-1 declines, the optimal quantity, holding cost, setup cost, carbon emission cost, quality improvement cost, advertisement cost, green investment technology cost, total cost, total profit all fall.

6. Managerial Implications

Defective production system with price-dependent demand and promotion is main topic of this article. Notwithstanding price strategy, demand for the items has an enormous effect. Results of sensitivity analysis are as follows.

1. Since they have a major impact on total profit, total cost, and total sales, the scaling parameter (b), the constant demand parameter (a) ought to be given priority during the decision-making process. Unit cost must be taken into account because it has a substantial impact on overall earnings.
2. Rising ordering expenses lead to higher ordering quantities and lower overall profitability. In these situations, it is recommended to buy in bulk to reduce delivery frequency and consequently raise ordering costs.
3. When maintenance costs increase, order volume, profit, and cycle time all decline. To prevent excessive holding expenses, it is recommended to only resupply what is required. Cycle time should also be decreased to ensure the smallest holding period possible.
4. SP rises as manufacturing expenses rise, but overall profit and order quantity fall. It is

advised to raise prices to preserve sales margin. It's also advised to cut back on orders and search for suppliers who provide reduced prices.

5. As the price of carbon emissions rises, both the overall profit and the amount of orders decrease. In order to minimize inventory holding, it is recommended to only order what is necessary from the manufacturer, as doing so may result in hazardous emissions.
6. As the frequency of ads increases, the system's overall profit also increases. Increased advertising frequency would surely draw larger audience, expand the potential customer base and increase overall profitability.
7. The management may benefit from increased demand if they strategically reduce or raise the SP to increase demand.
8. To increase profits, it is suggested that the retailer use the price break-even point. This method helps identify the price of the product at which there is no loss or profit. Based on this, retailer can then set a higher price depending on desired profit margin.

7.COMPARATIVE STUDY: COMPARATIVE RESEARCH IS CARRIED OUT AMONG SUBSTITUTE PRODUCTS AS WELL AS COMPLEMENTARY PRODUCTS, IS PRESENTED IN TABLE -8.

The table shows that complementary commodities perform better because they complement each other. This means such commodities are used together, increasing their worth. When consumed alone, these products are unused.

Table 8 Relationship between substitute products and complementary products

S.No.	Particulars	Substitute products	Complementary products
1.	Demand for the product	2676.44	2800.23
2.	Optimum quantity	127.62	215.69
3.	Optimal cycle time	0.8962	0.6706
4.	Production time	0.1276	0.2156
5.	Maximum inventory	108.35	144.85
6.	Setup cost	892.36	1192.92
7.	Production cost	284709.89	643256.63
8.	Holding cost	1083.58	1448.55
9.	Quality improvement cost	569.41	1286.51
10.	Advertisement cost	602.34	805.22

11.	Carbon emission cost	1191.94	1593.40
12.	Green Investment Technology cost	780.81	1043.01
13.	Total cost	289830.36	650627.06
14.	Total sales	377195.04	891627.34
15.	Total profit	87364.67	241000.28

8. CONCLUSIONS AND FUTURE STUDIES

Green production inventory model is developed for deteriorating items, incorporating advertisement-driven demand, Cournot's inverse demand, and sales returns. Decision variables include optimal demand, production lot size, formulated using a third-order equation. Two models have been proposed. In first model, a production inventory framework for perishable goods featuring Cournot's complementary products. In model 2, production inventory framework for perishable goods featuring Cournot's substitutable products. "Law of demand" is verified, price break-even point is defined, highest possible profit is calculated using 3 different product prices in two models. By identifying optimal pricing along with production lot size, this investigation seeks to maximize overall profit. Numerical examples demonstrate how this notion can be applied. It is demonstrated that the degree of complementarity and substitutability affects overall profit. Sensitivity analysis of best possibilities is presented to assess stability of proposed model. It uses Visual Basic 6.0 to create data. The complementing product's price break-even point is 2034.0423, whereas the substitute product's is 2045.1504. The demand for the complementary product is 2800.23, yielding a maximum possible profit of 2,41,000.28, while the highest possible demand for the substitute product is 2676.44, with a maximum profit of 2,89,800.36. From the sensitive analysis, total cost along with total profit are highly sensitive to increases in intercept of demand, as well as to changes in own-demand and cross-demand effects for both complementary and substitute products.

Food, stationery, bags, machines, and clothing (substitute products) with production concerns can be managed using the suggested inventory model. It applies to automobile (fuel), computer (software), printer (ink), etc. manufacturers.

Current research can be enhanced in the ways listed below:

- (i) Multi-stage production processes can be

included in this research since most systems currently in use are multi-stage production systems (Complementary analysis of a multi-item inventory model with leading product price by Kim et al. 2023).

- (ii) The lot transportation scenario in multi-stage manufacturing systems involves, among other things, delivery delays between stages. To further cut down on shipping expenses, the lot might be divided into smaller lots as well as sent as needed. An inventory model that takes scrap rework, rework into consideration could be developed with these scenarios in mind.
- (iii) The model can also be used for supply chain inventory (Jauhari et al., 2024, inventory model for two competing stores).
- (iv) Exponential demand, advertisement demand, fuzzy environments, probability demand, and stock-dependent demand (Hemmati et al. 2017; Koren et al. 2017; inventory for stock-based substitute products).
- (v) Carbon emissions and related carbon controls and limits may also be included in this study (carbon emissions regulations, Mahato et al. 2024)

Higher-order equations can be included in the study's limits. The price break-even amount does not take the fixed cost into account. Because of space limitations, competition among Seller 1 and 2 is not examined, despite the study's focus on a competitive market. This study does not account for transportation expenses. Future studies can examine each of these (transportation and pricing policies of complementary products by Dehghanbaghi and Sajadieh, 2017).

Declarations

Ethics approval to participate

We now disclose that neither animals nor human subjects are used in this study.

Consent for publication

We state that we have no known competing financial interests or personal relationships that could have influenced this paper's work or that could be

considered competing interests.

Availability of data and material

All of the information needed to support proposed model is included in manuscript.

Competing interests

We declare that we are not having known competing financial interests or personal relationships that might have influenced work presented in this paper. We also affirm that we have no financial interests//personal relationships that could be viewed as potential competing interests.

Funding

The authors were paid for the completion of this work. This research was conducted without the assistance of external funding.

Authors' contributions

V. Choudri, Senthilkumar, and C.K. Sivashankari contribute writing of paper, formating the mathematical models, generating data utilising visual basic 6.0, supervising the paper and writing the conclusion and reference.

Acknowledgements

It is acknowledged

REFERENCES

1. Asproudis, E., Filippiadis, E., (2021), Environmental Technological Choice in a Cournot-Bertrand Model," Journal of Industry, Competition and Trade, Vol. 21, No.1, pp. 43-58.
2. Benkherouf,L., Skouri, K., and Konstantaras,I., (2017), Inventory decisions for a finite horizon problem with product substitution options and time varying demand, Applied Mathematical Modelling, Vol. 51, pp. 669-685.
3. Bian, J., Lia, K.K. ,Zhao, X., Zhou, G., (2018), Bertrand vs. Cournot competition in distribution channels with upstream collusion, International Journal of Production Economics, Vol. 204, pp. 278-289.
4. Bischi, G.I., Lamantia, F., Radi, D., (2015), An evolutionary Cournot model with limited market knowledge, Journal of Economics Behavior & Organization, Vol. 116, pp. 219-238.
5. Chen, Y., Yang, L., Jiang, Y., Wahab, M., & Yang, J. (2019), Joint replenishment decision considering shortages, partial demand substitution, and defective items, Computers & Industrial Engineering, Vol. 127, pp. 420-435.
6. Chen, Z., and Su, S. I. I. (2021), Consignment supply chain cooperation for complementary products under online to offline business mode, Flexible Services and Manufacturing Journal, Vol. 33, No. 1, pp.136-182.
7. Choi, Chan (1991), Price competition in A Channel structure with A common retailer, Marketing Science, Vol. 10, pp. 271-295.
8. Coughlan, A. T. (1987), Distribution channel choice in a market with complementary goods, International Journal of Research in Marketing, Vol. 4, No.2, pp. 85-97.
9. De, M., Das, B., Maiti, M. (2016), EPL models for complementary and substitute items under imperfect production process with promotional cost and selling price dependent demands, OPSEARCH, Springer;Operational Research Society of India, Vol. 53, No. 2, pp. 259-277.
10. Dehghanbaghi, N., and Sajadieh, M.S., (2017), Joint optimization of production, transportation and pricing policies of complementary products in a supply chain, Computers & Industrial Engineering, Vol. 107, pp.150-157.
11. Drezner, Z., Gurnani, H., and Pasternack,B.A., (1995), An EOQ model with substitutions between products, Journal of Operations Research Society, Vol. 46, pp.887-891.
12. Edalatpour, M. A., & Al-e-Hashem, S. M. (2019), Simultaneous pricing and inventory decisions for substitute and complementary items with nonlinear holding cost. Production Engineering, Vol. 13, No. 3, pp.305-315.
13. Giri, R.N., Mondal, S.K., Maiti, M., (2016), Analysis of pricing decision for substitutable and complementary products with a common retailer, Pacific Sciene ReviewA : Natural Science and Engineering, Vol. 18, No.3, pp.190-202.
14. Gupta, V., Ivanov, D., Choi, T-M., (2021), Competitive pricing of substitute products under supply disruption, Omega, Vol. 101, No. 102279.
15. Haque, R., Pervin, M., Mondal, S.P., (2024), A sustainable production inventory model for two deteriorating complementary

- products under preservation technology and carbon cap regulation, Research Square Platform, doi.org/10.21203/rs.3.rs-4176730/v1
16. Hemmati, H., Ghomi, S.M.T., Sajadieh, M.S., (2017), Inventory of complementary products with stock-dependent demand under vendor managed inventory with consignment policy, *Scientia Iranica*, Vol. 25, No.4, pp.2347-2360.
 17. Jalali, H., Ansaripoor, A.H., Giovanni, P.E., (2020), Closed-loop supply chains with complementary products, *International Journal of Production Economics*, Vol. 229, Article No. 107757.
 18. Jauhari, W.H., Wangsa, I., D., Sofiana, A., Utama, D.M., (2024), An integrated inventory model for a supply chain system with two competing retailers, carbon emissions, and price-and-service dependent demand, *Sustainability Analytics and Modeling*, Vol. 4, Article No. 100029.
 19. Karmakar, S., De, S.K., Datta, T.K., Goswami, A., (2021), An Integrated Production Policy with Defective items and Stock-out based substitution under Triangular Dense Fuzzy Lock Set Environment, *RAIRO Operations Research*, Vol. 55, pp. S2727-S2746.
 20. Khan, M.A.A., Shaikh, A.A., Khan, A.R., Alrasheedi, A.F., (2023), Advertising and pricing strategies of an inventory model with product freshness-related demand and expiration date-related deterioration, *Alexandria Engineering Journal*, Vol. 73, pp. 353-375.
 21. Kim, S., Feng, Y., Xu, J., (2023), Complementary analysis of a Multi-item inventory model with leading product pricing, *Production and Operations Management*, Vol. 32, No 12, pp. 4190-4211.
 22. Koren, M., Perlman, Y., Shnaiderman, M., (2023), Inventory management for stockout-based substitutable products under centralised and competitive settings, *International Journal of Production Research*, Vol. 62, No.1, pp. 1-17.
 23. Koren, M., Perlman, Y., Shnaiderman, M., (2022), Inventory management for stockout-based substitutable products under centralised and competitive settings, *International Journal of Production Research*, Vol. 62, No.9, pp. 3176-3192.
 24. Koren, M., Perlman, Y., Matan Shnaiderman, M., (2022), Inventory Management Model for Stockout Based Substitutable Products, *IFAC Papers*, Vol. 55, No.10, pp. 613-618.
 25. Leonardos, C., (2020), Endogenizing the Cost Parameter in Cournot Oligopoly, *International Game Theory Review*, Vol. 22, No. 02, Article No. 2040004.
 26. Li, M., Chen, K., Wang, S., (2023), Retail outsourcing strategy in Cournot & Bertrand retail competitions with economies of scale, *Journal of Industrial and Management Optimization*, Vol. 19, No.1, pp.491-528.
 27. Mahato, F., Mahato, C. & Mahata, G.C. (2024), Sustainable inventory models with controllable deterioration under two-level trade credit policy and various carbon emissions regulations, *Modeling Earth System and Environment*, Vol. 10, pp.4449-4474.
 28. Maity, K., and Maiti, M. (2009), Optimal inventory policies for deteriorating complementary and substitute items, *International Journal of Systems Science*, Vol.40, No. 3, pp.267-276.
 29. Malcolm Brady, (2021), A Game-Theoretic Model of Strategic Interaction Using Advertising: Simulating the Evolution of the Cournot Nash Equilibrium under Different Competitive Scenarios, *Games*, Vol.12, No. 4, Article No. 85.
 30. McGillivray and Silver, E., (1978), Some concepts for inventory control under substitutable demand, *INFOR: Information Systems and Operation Research*, Vol. 16, pp.47-63.
 31. Meza, A., Latorre, P., Bonacic, M., Lopez-Ospina, H., Perez, J. (2024), Optimizing Inventory and Pricing for Substitute Products with Soft Supply Constraints, *Mathematics*, Vol. 12, No. 11, No. 1751.
 32. Mondal, C. and Giri, B.C. (2024), Pricing and bundling strategies for complementary products in a closed-loop green supply chain under manufacturers' different behaviors, *Expert Systems with Applications*, Vol. 238, Part E, No. 121960.
 33. Mukhopadhyay, A and Goswami, A., (2017), An inventory model with

- shortages for imperfect items using substitution of two product, *International Journal of Operational Research*, Vol. 30, pp.193-219.
34. Nielsen, E. and Moon, I., (2021), Strategic inventory and pricing decision for substitutable products, *Computers & Industrial Engineering*, Vol. 160, Article No 107570.
 35. Otake, T., and Min, K.J. (1996), Inventory and Pricing Policies for a Duopoly of Substitute Products, *Industrial Engineering Research Conference*, pp. 293-298.
 36. Parlar and Goyal, S., (1984), Optimal ordering decisions for two substitutable products with stochastic demands, *Opsearch*, Vol. 21, pp.1-15.
 37. Parlar, M. (1988), Game Theoretic Analysis of the Substitutable Product Inventory Problem with Random Demands, *Naval Research Logistics*, Vol. 35, pp. 397-409.
 38. Pasternack, A., and Drezner, Z., (1991), Optimal inventory policies for substitutable commodities with stochastic demand, *Naval Research Logistics*, Vol. 38, pp. 221-240.
 39. Perlman, Y., (2019), Inventory Levels of Two Stockout-based Substitutable Products under Centralized and Competitive Settings, *IFAC Papers*, Vol. 52, No. 13, pp. 1450-1454.
 40. Poormoaid, S., and Atan, Z., (2020), A continuous review policy for two complementary products with interrelated demand, *Computers & Industrial Engineering*, Vol. 150, Article No. 106980.
 41. Ren, M., Liu, J., Feng, S., Yang, A., (2020), Complementary Product Pricing and Service Cooperation Strategy in a Dual-Channel Supply Chain, *Discrete Dynamics in Nature and Society*, doi.org/10.1155/2020/2314659.
 42. Saha, S., Nielsen, I.E., Moon, I., (2021), Strategic inventory and pricing decision for substitutable products, *Computers & Industrial Engineering*, Vol. 160, Article No. 107570.
 43. Sarkar, B., Bar, N., Mukherjee, S.B., Sangal, I., Guchhait, R., (2024), Complementary Deteriorating Products within a Flexible Production System: A Sustainable Approach, *RAIRO Operations Research*, Vol. 58, pp.3823-3848.
 44. Sarkar, M., Lee, Y.H., (2017), Optimum pricing strategy for complementary products with reservation price in a supply chain model, *Journal of Industrial and Management Optimization*, Vol. 13, No.3, pp. 1553-1586.
 45. Shan, H., Zhang, C., and Wei, G. (2020), Bundling or unbundling? Pricing strategy for complementary products in a green supply chain, *Sustainability*, Vol. 12, No.4, Article No. 1331.
 46. Shocker, A.D.; Bayus, B.L.; Kim, N, (2004), Product compliments and substitutes in the real world: The relevance of "other products", *Journal of Marketing*, Vol. 68, pp. 28-40.
 47. Singh, N and Vives, X (1984), Price and quantity competition in a differentiated duopoly, *Rand Journal of Economics*, Vol. 15, pp.546-554.
 48. Taleizadeh, A. A., Babaei, M. S., Niaki, S. T. A., & Noori-daryan, M. (2020), Bundle pricing and inventory decisions on complementary products, *Operational Research*, Vol. 20, No. 2, pp. 517-541.
 49. Taleizadeh, A. A., Babaei, M. S., Sana, S. S., & Sarkar, B. (2019), Pricing decision within an inventory model for complementary and substitutable products, *Mathematics*, Vol. 7, No. 7, Article No. 568.
 50. Taleizadeh, A.A., and Charmchi, M., (2015), Optimal advertising and pricing decisions for complementary products, *Journal of Industrial Engineering International*, Vol. 11, pp. 111-117.
 51. Teunter, R.H. and Kuipers, S. (2022), Inventory control with demand substitution: new insights from a two-product Economic Order Quantity analysis, *Omega*, Vol. 113, Article No. 102712.
 52. Tian, Y., Lautz, S., Wallis, A.O.G., Lambiotte, R., (2021), Extracting complements and substitutes from sales data: a network perspective, *EPJ Data Science*, Vol. 10, No. 45.
 53. Varian, H.R. (1992), *Micro Economic Analysis*, New York, New York: W.W. Norton & Company.
 54. Wang, L., Song, H., Wang, Y., (2017), Pricing and service decisions of complementary products in a dual-channel supply chain, *Computers & Industrial Engineering*, Vol. 105, pp. 223-

- 233.
55. Wang, Q., Zhang., B, (2021), A guarantee credit model with substitutable product competition, *International Journal of Production Research*, Vol. 59, No. 22, pp. 6919-6940.
 56. Wang, Y. (2006), Joint pricing-production decisions in supply chains of complementary products with uncertain demand. *Operations Research*, Vol. 54, No. 6, pp. 1110-1127.
 57. Wei, J., Zhao, J., & Li, Y. (2013), Pricing decisions for complementary products with firms different market powers, *European Journal of Operational Research*, Vol. 224, No.3, pp. 507-519.
 58. Yue, X., Mukhopadhyay, S. K., & Zhu, X. (2006), A bertrand model of pricing of complementary goods under information asymmetry, *Journal of Business Research*, Vol. 59, No. (10–11), pp. 1182–1192.
 59. Zhao, J., Tang, W., Zhao, R., Wei, J., (2012), Pricing decisions for substitutable products with a common retailer in fuzzy environments, *European Journal of Operational Research*, Vol. 216, No. 2, pp. 409-419.